PARTICLE PHYSICS Phy 523
SOLUTIONS Midsemester -III

Attempt all questions; All questions carry equal marks.

April 9th 2009

1. (a) Assuming ν_μ's are left handed, what will be the helicity of μ^+ emited in π^+ decay $\pi^+ \to \mu^+ + \nu_\mu$ (in the rest frame of π)? (b) Write down the reaction of μ^- decay to electron and neutrinos. Let the μ^- be polarised along the positive z-axis and all the particles be emitted along the z- axis, with the electron moving along the negative z- axis and the neutrinos along the positive axis z- axis. Find the helicity of the electron. for this configuration.

Solution:

(a) π is a spin zero particle. and thus the spin of the final state must be zero. The two emitted particles μ^+ and ν_μ are emitted in opposite directions. Let ν_μ be travel along $z-$ axis. Then μ will travel along $-z$ direction. Since the helicity of ν_μ is negative its spin will be along $-z$ axis. Therefore the spin direction of μ^+ is along z-direction and it's helicity is negative.

(b) $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. The sum of the spins along the z- axis of the neutrinoes add to zero (ν_μ has negative helicity and $\bar{\nu}_e$ has positive helcity.) Thus the direction of spin of the elctron must be the same as that of the muon and is so the electron has spin along z and its helicity is negative.

2. Consider the lagrangian density for a complex scalar field $\Phi(x)$ given by

$$L(x) = (D^\mu \Phi)^\dagger(x) D_\mu \Phi(x) - \mu^2 \Phi^\dagger(x)\Phi(x) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

where $D^\mu \Phi(x) = (\partial^\mu + ieA^\mu)\Phi(x)$ and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, A^μ being the photon field. Show that the lagrangian density is invaraint under a transformation $\Phi'(x) = e^{i\alpha(x)}\Phi(x)$ provided A_μ is transformed appropriately. Find the transformation for A_μ under the above gauge transformatio

Solution:

$$D_\mu \Phi(x) = (\partial_\mu + ieA_\mu)\Phi(x)$$

Under a gauge transformation theis becomes

$$(\partial_\mu + ieA'_\mu)\Phi'(x) = (\partial_\mu + ieA'_\mu)e^{i\alpha(x)}\Phi(x)$$

$$= (e^{i\alpha(x)}\partial_\mu\Phi(x) + i\partial_\mu\alpha(x)e^{i\alpha(x)}\Phi(x) + ieA'_\mu)e^{i\alpha(x)}\Phi(x))$$

Thus if $ieA'_\mu(x) + i\partial_\mu\alpha(x) = ieA_\mu)(x)$ the above equation becomes

$$(\partial_\mu + ieA'_\mu)\Phi'(x) = e^{i\alpha(x)}(\partial_\mu + ieA_\mu)\Phi(x)$$

We also have

$$((\partial_\mu + ieA'_\mu)\Phi'(x))^\dagger = ((D^\mu\Phi)^\dagger(x))'$$

$$= ((D^\mu\Phi)^\dagger(x))e^{-i\alpha(x)}$$

thus the first term in the Langrangian becomes

$$(D^\mu\Phi)^\dagger(x)D_\mu\Phi(x)' = D^\mu\Phi)^\dagger(x)D_\mu\Phi(x)$$

showing that this term is invariant. The term

$$\mu^2\Phi'^\dagger(x)\Phi'(x) = \mu^2\Phi^\dagger(x)e^{-i\alpha(x)}e^{i\alpha(x)}\Phi(x) = \mu^2\Phi^\dagger(x)\Phi(x)$$

showing its invariance. The last term is $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ where $F^{\mu\nu} = \partial_\mu A_\nu - \partial_n u A_\mu$.So

$$F'^{\mu\nu} = \partial_\mu A'_\nu - \partial_n u A'_\mu$$

$$= \partial_\mu(A_\nu(x) - \frac{1}{e}\partial_\nu\alpha(x)) - \partial_\nu(A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x))$$

$$= \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) = F_{\mu\nu}(x)$$

Showing all the terms in the Langrangian are invariant. The transformation of $A_\mu(x)$ is given by

$$A'_\mu = a_m u(x) - \frac{1}{e}\partial_\nu\alpha(x)$$

3. Suppose we had a scalar doublet Δ in the standard model

$$\Delta = \begin{pmatrix} \delta^{++} \\ \delta^{+} \end{pmatrix}$$

What is the value of Y for the doublet?

Write down the covariant derivative for this doublet using $W_\mu^a \tau^a = W_\mu^a \sigma^a/2$ and B_μ as gauge fields with coupling g, g' respectively. Introducing the photon field A_μ and the Z-boson field Z_μ, defined by

$$W_\mu^3 = \frac{g' A_\mu + g Z_\mu}{(g^2 + g'^2)^{1/2}}, B_\mu = \frac{g A_\mu - g' Z_\mu}{(g^2 + g'^2)^{1/2}}$$

show that the coupling to photon is to the charge of the particle with $e = gg'/(g^2 + g'^2)^{1/2}$.

We know $D_\mu = \partial_\mu - igW_\mu^a \sigma^a/2 - ig'YB_\mu$ In the case of Δ we have $T = 1/2$ and $Y = 3/2$ Thus the covariant derivative for the doublet is

$$D_\mu \Delta = (\partial_\mu - igW_\mu^a \frac{\sigma^a}{2} - \frac{3ig'}{2} B_\mu)\Delta$$

Consider

$$gW_\mu^3 T^3 + g'YB_\mu = g\frac{g' A_\mu + g Z_\mu}{(g^2 + g'^2)^{1/2}} T^3 + g'Y \frac{g A_\mu - g' Z_\mu}{(g^2 + g'^2)^{1/2}}$$

$$= \frac{gg'}{(g^2 + g'^2)^{1/2}}(T^3 + Y)A_\mu + \frac{g^2 T^3 - g'^2 Y}{(g^2 + g'^2)^{1/2}} Z_\mu$$

Using $Q = T^3 + Y$ we get for the coupling of the electromagnetic field

$$\frac{gg'}{(g^2 + g'^2)^{1/2}} Q A_\mu = eQA_\mu$$

where

$$e = \frac{gg'}{(g^2 + g'^2)^{1/2}}$$

is the elctric charge.