Phy 523 FINAL (SOLUTIONS)

Final Examination

The paper has two parts. Questions in the part I is for five marks each. Questions of part II is of ten marks each. Attempt six questions from part I and three from part II.

April 21st 2009

Time allowed 3 Hours

PART I

1. Describe (BE BRIEF) an experiment which shows that the law of parity conservation is not valid in weak interactions.

Solution: Consider the beta decay $^{60}Co \rightarrow ^{60} Ni^{*} + e^{-}\bar{\nu}_e$ with the spin of the Cobalt nuclues alligned along z-derection. (Can be done using a magnetic field) The electrons are emitted asymmetrically with more electrons emitted in the backward direction than the forward direction. This shows that Parity is violated in the decay. To see this note the spin vector $\vec{S}$ is an axial vector and hence $\vec{S} \rightarrow \vec{S}$ under parity. However the momentum vector$\vec{P}_e$ of the electron is a polar vector and we have it transform under parity as $\vec{P}_e \rightarrow -\vec{P}_e$. Thus if electrons are preferentially emitted in the backward direction we have more electrons emitted in the forard direction in the parity transformed "experiment". This shows parity is violated.

2.A particle of mass $1000MeV/c^2$ decays to two pions with a decay rate of $\Gamma \approx 100MeV$. Is this decay due to weak, electromagnetic or strong interaction? Justify your answer.

Solution:

We can write $\Gamma \approx \frac{g^2}{4\pi}M$ so this leads to

$$\frac{g^2}{4\pi} \approx \frac{1}{10}$$

and hence is due to strong interaction.

3. Consider the reaction

$$\pi^{-} + p \rightarrow \Lambda^0 + K^0$$

. Find the minimum energy in the laboratory frame (proton at rest) for the above reaction to occur. The rest masses of the particles are m_π, m_p, m_Λ and m_K. Note $m_\Lambda + m_K > m_p + m_\pi$.

Solution:
The minimum of $s = (P_\Lambda + p_K)^2$ is $(m_\Lambda + m_K)^2$ and so

$$(p_\pi + p_p)^2 = m_\pi^2 + m_p^2 + 2m_p E_\pi = (m_\Lambda + m_K)^2$$

for the minimum value of E_π. Thus

$$E_\pi(min) = \frac{(m_\Lambda + m_K)^2 - m_\pi^2 - m_p^2}{2m_p}$$

4. Consider a massive spin one particle moving along the 3-axis with momentum P. If it is polarised with spin component 0 along the 3rd axis, write down all the four components of the polarisation vector.(Mass=M) (Use $(S^i)_{jk} = -i\epsilon_{ijk}$ for the ith component of spin S^i.)

Solution:
We have $P_\mu \epsilon^\mu = 0$. In the rest frame this implies $\epsilon^0 = 0$ and $(S^3)_{ij}\epsilon^j(\vec{P} = 0) = 0$, we have $\epsilon^1(\vec{P} = 0) = \epsilon^2(\vec{P} = 0) = 0$ and $\epsilon^3(\vec{P} = 0) = 1$. Since the particle is moving along the third axis,we have for the components of the polarisation vector in this frame $\epsilon^1 = \epsilon^2 = 0$. Further let $\epsilon^0 = a; \epsilon^3 = b$; we have $a^2 - b^2 = 0$ and $aP^0 - Pb = 0$; where $P = |\vec{P}$. This gives

$$\epsilon^\mu = (-P, 0, 0, P^0)/M$$

(The overall sign is not relevent. It has been fixed by the convention)

5. Show that the concept of helicity is not a Lorentz invariant for massive particles.

Solution: Let the particle travel with a velocity v in some direction and let the spin point in the same direction (Positive helicity) If a Lozentz transformation is made along the direction of v with a speed $u > v$ the the paricle would backward and its spin will point in the same direction. This would be a negative helicity particle. This can be done only if the particle is massive as u can exceed v only if $v < c$.

6.Suppose a Langrangian is invariant under $SU(3)$ and it undergoes spontaneous breaking to $SU(2)$.

(a) How many generators do $SU(3)$ and$SU(2)$ have?
(b) How many Goldstone bosons appear in the theory ?

Solution:
(a)$SU(N)$ has N^2-1 generators and hence $SU(3)$ has 8 generators and $SU(2)$ has 3.
(b) the number of Golstone bosons $=8-3=5$.

7.How does the ratio $R=\sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ vary as the centre of mass energy of e^+e^- varies from 1Gev to 20GeV.

Solution: Hadrons are produce by fragmentation of quarks. We have $u.d.s.c.b$ quarks which can be produced in the energy range. As quarks have colour we have for the ration of creoss sections

$$R=3\sum_i Q_i^2$$

For energy less than 3 GeV onle u,d and s - quarks contribute and so

$$R=3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9})=2$$

For enrgy in the range 3 and 10 GeV c-quark also contributes and we have

$$R=3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9})=\frac{10}{3}$$

In the energy range 10-20-Gev b- quark also contributes and we have

$$R=3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9})=\frac{11}{3}$$

8.Using genralised statisitics (interchange in space,spin and isospin space) find the allowed combinations of I, S and L for a bound state of a neutron and a proton.

Solution:
Neutron, Proton being spin half particle obey Fermi-Dirac statistics. Thus the total wave function (orbital, spin and isospin) must be antisymmetric under the exchange of the two particles. $I=1$ is symmetric. $S=1$ is symmetric; L =even is symmetric. $I=0$ is anisymmetric,$S=0$ is antisymmetric and L=odd is antisymmetric under exchange. Thus the allowed

combinations are L=even and $S = 1, I = 0$ or $S = 0, I = 1$. Or L=odd and $S = 1, I = 1$ or $S = 0, I = 0$.

PART II

9. Consider the reaction $\pi^+(p_1)+\pi^+(p_2) \rightarrow \pi^+(q_1)+\pi^+(q_2)$ whose matrix element is given by (momenta indicated in brackets)

$$M = N\lambda(2\pi)^4\delta^4(p_1 + p_2 - q_1 - q_2)$$

where N is the normalisation constant and λ is the coupling constant. Calculate the total cross section in the centre of mass frame. If the total cent re of mass energy $E = 1GeV$ calculate the cross section in cm^2 (in terms of λ; mass of pion $= 140MeV/c^2$.)

Solution:

$$\frac{|M|^2}{VT} = |N|^2|\lambda|^2(2\pi)^4\delta^4(p_1 + p_2 - q_1 - q_2)$$

The flux in the centre of mass frame is $2|\vec{p}_1|/p_1^0$. Thus

$$\sigma = \frac{1}{2|\vec{p}_1|/2p_1^0}\frac{1}{(2\pi)^2}\int\frac{d^3q_1d^3q_2}{(16p_1^0p_2^0q_1^0q_2^0)}$$

$$= \frac{\lambda^2}{(2\pi)^2}\int\frac{|\vec{q}_1|^2d|\vec{q}_1|d\Omega_1}{32p_2^0q_1^0q_2^02|\vec{p}_1|}\delta(p_1^0 + p_2^0 - q_1^0 - q_2^0)$$

performing the Ω integral which gives 4π we get

$$\sigma = \frac{\lambda^2}{\pi}\int\frac{|\vec{q}_1|q_1^0d|q_1^0}{32q_1^0q_2^0|\vec{p}_1|p_2^0}\delta(2p_1^0 - 2q_1^0)$$

where we have used $q_1^0d|q_1^0 = |\vec{q}_1|d|\vec{q}_1|$ and $q_1^0 = q_2^0 = p_1^0 = p_2^0$ in the centre of mass frame. We also use the Mandelstam variable $s = (p_1 + p_2)^2 = 4(p_1^0)^2$ Doing the q_1^0 integral gives a factor $1/2$ due to the δ-function and we have

$$\sigma = \frac{\lambda^2}{64\pi(p_1^0)^2} = \frac{\lambda^2}{16\pi s}.$$

We can evaluate the cross section at $s = 1GeV^2$ and we have

$$\sigma = \frac{\lambda^2}{16\pi}\frac{\hbar^2c^2}{(1.610^{-19}10^9)^2}$$

$$= \frac{\lambda^2}{4\pi} 10^{-28} cm^2$$

10. Write the matrix element for the μ decay $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$ in the standard model. Show that in the limit $m_W >> m_\mu$, it goes over to the "current current" matrix element $iN(2\pi)^4\delta^4(P_\mu - P_e - P_{\bar{\nu}_e} - P_{\nu_\mu})\frac{G_F}{\sqrt{2}}\bar{u}_{\nu_\mu}\gamma_\mu(1-\gamma_5)u_\mu\bar{u}_e\gamma^\mu(1-\gamma_5)v_{\nu_e}$. Write down the relation between G_F and the $SU(2)_L$ coupling g.

Solution: The matrix element in the standard model is

$$M = (\frac{g}{2\sqrt{2}})^2 N\bar{u}_{\nu_\mu}(P_{\nu_\mu}\gamma^\lambda u_\mu(P_\mu)(-i)\left(\frac{g_{\lambda\beta} - \left(\frac{q_\lambda q_\beta}{m_W^2}\right)}{q^2 - m_W^2}\right)\bar{u}_e(P_e)\gamma^\beta v_{\bar{\nu}_\mu}(P_{\bar{\nu}_e}$$

where $q = P_\mu - P_{\nu_\mu}$. The term $\frac{q_\lambda q_\beta}{m_W^2}$ does not contribute when we neglect the masses of the leptons.

For $q^2 << m_W^2$, we get for the matrix element

$$M = (\frac{g}{2\sqrt{2}})^2 N\bar{u}_{\nu_\mu}(P_{\nu_\mu}\gamma^\lambda u_\mu(P_\mu)(-i)\frac{g_{\lambda\beta}}{-m_W^2}\bar{u}_e(P_e)\gamma^\beta v_{\bar{\nu}_\mu}(P_{\bar{\nu}_e})$$

$$= (\frac{ig^2}{8m_W^2})N\bar{u}_{\nu_\mu}(P_{\nu_\mu})\gamma^\lambda u_\mu(P_\mu)\bar{u}_e(P_e)\gamma^\lambda v_{\bar{\nu}_\mu}(P_{\bar{\nu}_e})$$

Comparing it with $i\frac{G_F}{\sqrt{2}}\bar{u}_{\nu_\mu}\gamma_\mu(1-\gamma_5)u_\mu\bar{u}_e\gamma^\mu(1-\gamma_5)v_{\nu_e}$ we have

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

11. (a) Consider the reaction $A + B \rightarrow C + D$ in the centre of mass frame with the total initial energy $2(M_C + M_D)$. Find the energy of the outgoing particles C and D. (Masses of the particles A, B, C and D are M_A, M_B, M_C, M_D respectively.)

(b) If a particle is unstable with a decay rate Γ this can be phenomenologically represented by adding an imaginary part to the mass term. Justify the above statement and write down the appropriate expression for the mass term including the complex part.

Solution:

(a)In the centre of mass frame we have $P_T = P_A^\mu + P_B^\mu = (2(m_A + m_B), \vec{0})$ We also have $P_T = P_C + P_D$ Writing $P_T - P_C = P_D$ and squaring both sides we get

$$4(m_C + m_D)^2 + m_C^2 - 2(2(m_C + m_D))E_C = m_D^2$$

or

$$E_C = \frac{5m_C^2 + 3m_D^2 + 8m_C m_D}{4(m_C + m_D)} = \frac{5m_C + 3m_D}{4}$$

(b) $Number(t) \propto e^{-\Gamma t}$ so we write for the for the matrix element

$$M \propto e^{im - \Gamma t/2}$$

leading to

$$|M|^2 \propto e^{-\Gamma} t$$

and we add in the mass an imaginary term

$$m - i\frac{\Gamma}{2}$$

which accounts for the decay.

12. (a) Derive the relation

$$\Sigma_{r=1,2} u(p,r)\bar{u}(p,r) = (\not{p} + m)$$

where $u(p,r), r = 1,2$ represents positive energy wavefunctions of the free Dirac equation and m is the mass of the particle.

(b) Consider a spin zero particle which is an eigenstate of the charge conjgation operator C. Asuume it to be made of quark-antiquark pair. Show that the C-parity must be positive.

Solution:

(a) Multiply both sides by $u(p,s)$. using the relation $\bar{u}(p,r)u(p,s) = 2m\delta_{rs}$ we have for the left hand side

$$\Sigma_{r=1,2} u(p,r)\bar{u}(p,r)u(p,s) = 2mu(p,s)$$

The right hand side gives the same result as $\not{P}u(p,s) = mu(p,s)$

$$(\not{p} + m)u(p,s) = 2mu(p,s)$$

Since this is true for any spinor with momentum p, we have the identity

$$\Sigma_{r=1,2} u(p,r)\bar{u}(p,r) = (\not{p} + m)$$

(b) $C = (-1)^{L+S}$ Since $J + 0$, $L + S$ and hence $C + (-1)^{2L} = +1$.