Phy 523 PARTICLE PHYSICS
SOLUTIONS MIDSEMESTER I

Q1. If the decay rate of $\Delta^{++} = \Gamma^{++}$ what is the decay rate of Δ^{+}?

Solution: In the decay isospin is conserved. Since $I = 3/2$ for the Δ the final state can only be $I = 3/2$. The decay Of Δ^{+} can occur through the channels

$$\Delta^{=} \rightarrow p + \pi^0$$

and

$$\Delta^{=} \rightarrow n + \pi^+$$

We have and

$$|n\pi^+ >= \left(\frac{1}{3}\right)^{1/2} |I = 3/2, I_z = 1/2 > + \left(\frac{2}{3}\right)^{1/2} |I = 1/2, I_z = 1/2 >$$

$$|p\pi^0 >= \left(\frac{2}{3}\right)^{1/2} |I = 3/2, I_z = 1/2 > + \left(\frac{1}{3}\right)^{1/2} |I = 1/2, I_z = 1/2 >$$

Thus

$$< n\pi^+|M|\Delta^+ >= \left(\frac{1}{3}\right)^{1/2} < I = 3/2, I_z = 1/2|M|\Delta^+ >$$

$$< p\pi^0|M|\Delta^+ >= \left(\frac{2}{3}\right)^{1/2} < I = 3/2, I_z = 1/2|M|\Delta^+ >$$

Hence

$$\Gamma^+(n\pi^+) = \frac{1}{3}\Gamma^{++}$$

$$\Gamma^+(p\pi^0) = \frac{2}{3}\Gamma^{++}$$

Total rate is

$$\Gamma^+ = \Gamma^+(n\pi^+) + \Gamma^+(p\pi^0) = \Gamma^{++}$$

2. A particle X of mass M_X decays to two paricls Y (mass M_Y). The decay distribution is isotropic in the rest frame of X. Consider a frame in which X is travelling with energy E_X along the 3rd direction. Show that the

energy distribution in the moving frame is constant- that is if dN_Y is the number emitted in the energy range E_Y to $E_Y + dE_Y$ show that

$$\frac{dN_Y}{dE_Y} = constant$$

and

$$\frac{\gamma}{2}[M_X - \beta\sqrt{(M_X^2 - 4M_Y^2)}] < E_Y < \frac{\gamma}{2}[M_X + \beta\sqrt{(M_X^2 - 4M_Y^2)}$$

where $\gamma = E_X/M_X$ and $\beta = \sqrt{(E_X^2 - M_X^2)}/E_X$

Solution:

Let the momentum four vector of Y in the rest frame of X be $(E_Y(r), P_Y(r)sin(\theta), 0, P_Y(r)cos(\theta))$ θ is the angle made by the momentum vector of Y in the rest frame of X. Here $E_Y(r) = m_X/2; p_Y(r) = (m_X^2/4 - m_Y^2)^{1/2}$. We can get the energy in the frame in which X is moving by a Lorentz transformation

$$E_Y = \gamma(E_Y(r) + \beta P_Y(r)cos(\theta))$$

where $\gamma = E_X/m_X; \beta = (E_X^2 - m_X^2)^{1/2}/E_X$. This implies (using $E_Y(r) = m_X/2; p_Y(r) = (m_X^2 - 4m_Y^2)^{1/2}/2$

$$\frac{\gamma}{2}[M_X - \beta\sqrt{(M_X^2 - 4M_Y^2)}] < E_Y < \frac{\gamma}{2}[M_X + \beta\sqrt{(M_X^2 - 4M_Y^2)}$$

Thus $\frac{dE_Y}{dcos(\theta)} = \gamma\beta P_Y$ and

$$\frac{dN}{dE_Y} = \frac{dN_Y}{dcos(\theta)}\frac{dcos(\theta)}{dE_Y} = \frac{1}{\gamma\beta} \times constant$$

as $\frac{dN_Y}{dcos(\theta)}$is a constant for isotropic distribution.

Q3. A ρ^0 meson is in the rest frame in the eigenstate of spin operator S_3 with eigenvalue 0. Write the matrix element for the decay of ρ^0 to $\pi^+\pi^-$. Find the angular distribution of the decay products. (You can use the matrix element

$$M = g\epsilon^\mu(p_{\pi^+} - p_{\pi^-})_\mu$$

where g is a constant.) ϵ^μ represents the polarization vector of ρ^0 meson and p_{π^+}, p_{π^-} are the four momenta of the π^+, π^- respectively.

Solution:

In the notation for the spin operators $(S_i)_{jk} = -i\epsilon ijk$ we have the eigenvector for S_3 with eigenvalue zero is

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Thus $\epsilon^0 = \epsilon^1 = \epsilon^2 = 0; \epsilon^3 = 1$. the matrix element for ρ- decay becomes

$$M = g(p_{\pi^+} - p_{\pi^-})_3 = -2gp^3_{\pi^+} = -2g|p_{\pi^+}|cos(\theta)$$

where $cos(\theta)$ is the angle with restect to the spin fo the ρ. We are in the rest frame of ρ and so $p_{\pi^+3} = -p_{\pi^-3}$. The angular distribution $\propto |M|^2 \propto cos^2(\theta)$