Phy 523 PARTICLE PHYSICS
Solution Midsemester -II

Attempt all questions; All questions carry equal marks.

March 28th 2009
Time allowed 90 minutes

1. Consider the decay of $\Lambda^0(P_\Lambda) \rightarrow p(P_p) + \pi^-(P_\pi)$ whose matrix element is given by

$$< pP_p; \pi P_\pi|M|\Lambda^0 P_\Lambda >= N\bar{u}(P_p)(A + B\gamma_5)u(P_\Lambda)(2\pi)^4\delta^4(P_\Lambda - P_p - P_\pi)$$

where N is the normalisation constant.

Show that the terms $\bar{u}(P_p)\sigma_{\alpha\beta}P_p^\alpha P_\pi^\beta u(P_\Lambda)$ and $\bar{u}(P_p)\sigma_{\alpha\beta}P_\pi^\alpha P_\Lambda^\beta u(P_\Lambda)$ can be converted to the terms of the form A and B. ($\sigma_{\alpha\beta} = i(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)/2$).

Solution:

We have $P_\Lambda = P_p + P_\pi$. So

$$\bar{u}(P_p)\sigma_{\alpha\beta}P_p^\alpha P_\pi^\beta u(P_\Lambda)$$

$$= \bar{u}(P_p)\sigma_{\alpha\beta}P_p^\alpha(P_\Lambda^\beta - P_p^\beta)u(P_\Lambda)$$

$$= \bar{u}(P_p)\frac{i}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)P_p^\alpha(P_\Lambda^\beta - P_p^\beta)u(P_\Lambda)$$

$$= \bar{u}(P_p)\frac{i}{2}(\not{P}_p \not{P}_\Lambda - \not{P}_\Lambda \not{P}_p)u(P_\Lambda)$$

as $\sigma_{\alpha\beta}P_p^\alpha P_p^\beta = 0$. Using the identity $\not{P}_\Lambda \not{P}_p = -\not{P}_p \not{P}_\Lambda + 2P_\Lambda.P_p$, we get

$$\bar{u}(P_p)\sigma_{\alpha\beta}P_p^\alpha P_\pi^\beta u(P_\Lambda)$$

$$= baru(P_p)\frac{i}{2}(2 \not{P}_p \not{P}_\Lambda - P_\Lambda.P_p)u(P_\Lambda)$$

We noe use $\not{P}_\Lambda u(P_\Lambda) = m_\Lambda u(P_\Lambda)$ and $\bar{u}(P_p) \not{P}_p = \bar{u}(P_p)m_p$ to get

$$\bar{u}(P_p)\sigma_{\alpha\beta}P_p^\alpha P_\pi^\beta u(P_\Lambda)$$

$$= baru(P_p)\frac{i}{2}(2m_p m_\Lambda - 2P_\Lambda.P_p)u(P_\Lambda)$$

which is of the form$\bar{u}(P_p)Au(P_\Lambda)$ with $A = \frac{i}{2}(2m_p m_\Lambda - 2P_\Lambda.P_p)$ The term $baru(P_p)\sigma_{\alpha\beta}P_\pi^\alpha P_\Lambda^\beta u(P_\Lambda)$ can be written as (using the conservation of momentum)

$$baru(P_p)\frac{i}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)(-P_p^\alpha P_\Lambda^\beta)u(P_\Lambda)$$

here we have dropeed the term $\frac{i}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)(P_\Lambda P_\Lambda)$ as this $= 0$. This can be simplified to

$$baru(P_p)\frac{i}{2}(- \not{P}_p \not{P}_\Lambda + \not{P}_\Lambda \not{P}_p)u(P_\Lambda)$$

$$= baru(P_p)\frac{i}{2}(-2m_p m_\Lambda + 2 + 2P_\Lambda.P_p)u(P_\Lambda)$$

This is of the form $\bar{u}(P_p)Au(P_\Lambda)$ with $A = \frac{i}{2}(-2m_p m_\Lambda + 2P_\Lambda.P_p)$.

kip 3mm 2. Consider the interaction of a Dirac particle $\Psi(x)$ with a scalar field $\phi(x)$ obeying the equation

$$(i \not{\partial} - m)\Psi(x) = -g\phi(x)\gamma_5\Psi(x)$$

Show that

$$\Psi_i(x) = \psi_i(x) - g\int d^4y S_F(x-y)\phi(y)\gamma 5\Psi_i(y)$$

where $S_F(x-y)$ is the free particle Feynman propagator.Ψ

Show that S-matrix element is given by

$$S_{fi} = (2\pi)^3\delta_{if} + ig\epsilon\int d^4y\bar{\psi}_f(y)\phi(y)\gamma_5\Psi_i(y)$$

where $\psi_i(x), \psi_f(x)$ are the free particle initial and final wave functions.$\epsilon = (-1)^n$ where n is the number of antiparticle at time $-\infty$.

(You can use the expression for the Feynman propagator

$$S_F(x-y) = -i\theta(x^0-y^0)\int \frac{d^3p}{(2\pi)^3}\sum_{r=1,2}\psi_p^r(x)\bar{\psi}_p^r(y) + i\theta(y^0-x^0)\int \frac{d^3p}{(2\pi)^3}\sum_{r=3,4}\psi_p^r(x)\bar{\psi}_p^r(y)$$

derived in the class. $\psi_p^r(x)$ are the plane wave solutions.)

2. $S_F(x-y)$ is defined as

$$(i \not{\partial} - m)S_F(x-y) = \delta^4(x-y)$$

Thus

$$(i\not\partial - m)\Psi_i(x) = (i\not\partial - m)\psi_i(x)$$
$$-(i\not\partial - m)g\int d^4y S_F(x-y)\phi(y)\gamma 5\Psi_i(y)$$
$$= -g\int d^4y(i\not\partial - m)S_F(x-y)\phi(y)\gamma 5\Psi_i(y)$$
$$= -g\int d^4y\delta^4(x-y)\phi(y)\gamma 5\Psi_i(y)$$
$$= -g\phi(x)\gamma_5\Psi_i(x)$$

we have used $(i\not\partial - m)\psi_i(x) = 0$ as $\psi_i(x)$ satisfies the free particle Dirac eqaution

$$\Psi_i(x) = \psi_i(x) - g\int d^4y(-i\theta(x^0-y^0)\int \frac{d^3p}{(2\pi)^3}\sum_{r=1,2}\psi_p^r(x)\bar{\psi}_p^r(y) + i\theta(y^0-x^0)\int \frac{d^3p}{(2\pi)^3}\sum_{r=3,4}\psi_p^r(x)\bar{\psi}_p^r(y)\phi($$

For $x^0 > y^0$, we get for $S_{fi} = \int d^3x\psi_f^\dagger(x)\Psi_i(x)$

$$= \int d^3x\psi_f^\dagger(x)\psi_i(x) - ig\int \frac{d^3p}{(2\pi)^3}\sum_{r=1,2}\int d^3x\psi_f^\dagger(x)\psi_p^r(x)\bar{\psi}_p^r(y\gamma_5\Psi_i(y)$$

The first term is (remembering $\psi_i(x) = \frac{1}{(2p_i^0)^{1/2}}u(p_i)e^{-ip_i.x}$

$$\int d^3x\frac{1}{4p_f^0p_i^0}u(p_f)^\dagger u(p_i)e^{i(p_f-p_i).x}$$

$$= (2\pi)^3\frac{1}{4p_f^0p_i^0}u(p_f)^\dagger u(p_i)\delta^3(\vec{p}_f - \vec{p}_i)e^{i(p_f^0-p_i^0)x^0}$$

Since $\vec{p}_f = \vec{p}_i$ because of the δ-function,$p_f^0 = p_i^00$. This implies the four vectors p_f and p_i are equal. Thus the exponential factor $e^{i(p_f^0-p_i^0)x^0} = 1$. Using $u(p)^\dagger u(p) = 2p^0$, we get

$$\int d^3x\frac{1}{4p_f^0p_i^0}u(p_f)^\dagger u(p_i)e^{i(p_f-p_i).x} = (2\pi)^3\delta^3(p_f - p_i) = (2\pi)^3\delta_{if}$$

The second term again contains the integral

$$\int d^3x\psi_f^\dagger(x)\psi_p^r(x) = (2\pi)^3\delta^3(p_f - p)\delta fr$$

thus we can perform the p- integration and the summation over r in the second term and obtain

$$+ig\epsilon \int d^4y \bar{\psi}_f(y)\phi(y)\gamma_5\Psi_i(y)$$

If x^0, y^0 the second term in S_F contributes which has a negative sign compared to the first term. The calculation is identical to what has been done for the case of $x^0 > y^0$, except for the sign. This explains the presence of ϵ in the final expression for the S-matrix.