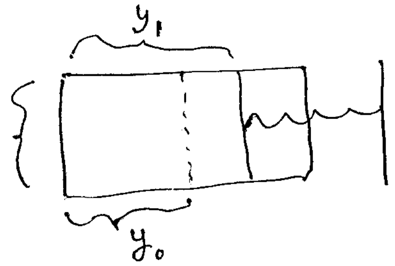
Problem Sheet I   S. Mech. Aug - Dec 2019

1

Area = A



place of piston for unstretched / or compressed spring.

$$10^3 (200 A - 100 A) = k(y_1 - y_0)$$

$$10^3 (500 A - 100 A) = k(2y_1 - y_0)$$

$$= ky + 10^3 (100) A$$

$$k y_1 = (300 \text{ kPa}) A$$

$$100 \text{ kPa } A = (300 \text{ kPa}) A - k y_0$$

$$k y_0 = 200 \text{ kPa} \times A$$

$$A y_1 = 0.1 \text{ m}^3$$

$$W.D = A\int_{y_1}^{2y_1} P(y)\, dy .$$

$$A[P(y) - 100] = k(y - y_0) = ky - 200 \text{ kPa } A$$

$$A P(y) = ky - 100 \text{ kPa } A$$

$$W.D = k\int_{y_1}^{2y_1} y\, dy - 100 \text{ kPa } A [y_2 - y_1]$$

$$= k \cdot \left[\frac{3y_1^2}{2}\right] - 100 \text{ kPa } y_1 A$$

$$= \frac{3}{2} \times 300 \text{ kPa } A y_1 - 100 \text{ kPa } A y_1 .$$

$$= (350 \text{ kPa}) A y_1 = 35 \text{ kPa.m}^3 .$$

2. $$\left.\frac{\partial F}{\partial x}\right)_{y,z} dx + \left.\frac{\partial F}{\partial y}\right)_{x,z} dy + \left.\frac{\partial F}{\partial z}\right)_{x,y} dz = 0$$

If z is Constant

$$\left.\frac{\partial F}{\partial x}\right)_{y,z} = -\left.\frac{\partial F}{\partial y}\right)_{x,z} \left.\frac{\partial y}{\partial x}\right)_{z}$$

$$\left.\frac{\partial y}{\partial x}\right)_{z} = -\frac{\left.\frac{\partial F}{\partial x}\right)_{y,z}}{\left.\frac{\partial F}{\partial y}\right)_{x,z}}$$

Also $$\left.\frac{\partial F}{\partial x}\right)_{y,z} \left.\frac{\partial x}{\partial y}\right)_{z} = -\left.\frac{\partial F}{\partial y}\right)_{x,z}$$

or $$\left.\frac{\partial x}{\partial y}\right)_{z} = -\frac{\left.\frac{\partial F}{\partial y}\right)_{x,z}}{\left.\frac{\partial F}{\partial x}\right)_{y,z}} = \frac{1}{\left.\frac{\partial y}{\partial x}\right)_{z}}.$$

$$\left.\frac{\partial y}{\partial x}\right)_{z} = -\frac{\left.\frac{\partial F}{\partial x}\right)_{y,z}}{\left.\frac{\partial F}{\partial y}\right)_{x,z}} \; ; \; \left.\frac{\partial z}{\partial y}\right)_{x} = -\frac{\left.\frac{\partial F}{\partial y}\right)_{x,z}}{\left.\frac{\partial F}{\partial z}\right)_{x,y}}$$

$$\left.\frac{\partial y}{\partial x}\right)_{z} \left.\frac{\partial z}{\partial y}\right)_{x} = \frac{\left.\frac{\partial F}{\partial x}\right)_{y,z}}{\left.\frac{\partial F}{\partial z}\right)_{x,y}} = -\left.\frac{\partial z}{\partial x}\right)_{y} = -\frac{1}{\left.\frac{\partial x}{\partial z}\right)_{y}}$$

or $$\left.\frac{\partial y}{\partial x}\right)_{z} \left.\frac{\partial z}{\partial y}\right)_{x} \left.\frac{\partial x}{\partial z}\right)_{y} = -1$$

3. $$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

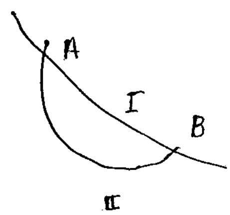
$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\int_{V_1}^{V_2} p\,dv = RT \ln \frac{V_2 - b}{V_1 - b} + a\left[\frac{1}{V_2} - \frac{1}{V_1}\right]$$

4 If we make a cycle from I → II and back

heat is converted to work in I and no heat is exchanged in II as it is adiabic.

Violation of II law.



5. $$E_1 < \frac{p_1^2}{2m} < E_2 \qquad \sqrt{2mE_1} < |p_1| < \sqrt{2mE_2}$$

Phase space : $2L\left[\sqrt{2mE_2} - \sqrt{2mE_1}\right]$