Phy 523
PARTICLE PHYSICS
Problem sheet VII

3rd March 2009

10th March 2009

31. Consider the spin zero particle under the action of a scalar current J obeying the equation

$$\partial^\mu \partial_\mu \Phi(x) + m^2 \Phi(x) = -J(x)$$

Writing the Greens function as

$$\left(\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x_\mu} + m^2\right)\Delta_F(x,y) = -\delta^4(x-y)$$

show that the general solution of the spin one equation in the presence of J is

$$\Phi_i(x) = \phi_i^0 + \int d^4z \Delta_F(x-z) J(z)$$

where ϕ^0 is the solution of the free Klein Gordon equation.

32. Evaluate the propator $\Delta_F(x-y)$ using the Feynman boundary condition and show it can be written as

$$\Delta_F(x-y) = -i\theta(x_0 - y_0)\int \frac{d^3p}{(2\pi)^3} f_p^+(x) f_p^{(+)*}(y)$$

$$-i\theta(y_0 - x_0)\int \frac{d^3p}{(2\pi)^3} f_p^-(x) f_p^{(-)*}(y)$$

. where

$$f_p^+(x) = \frac{1}{\sqrt{2p^0}} e^{-ip.x}; \quad f_p^-(x) = \frac{1}{\sqrt{2p^0}} e^{ip.x}$$

33. Let $J(x) = g\bar{\psi}(x)\psi(x)$ where is a spin half field and g is the coupling constant. Ψ obeys the equation

$$(i \not{\partial} - m)\Psi(x) = -g\phi(x)\Psi(x) ... \quad Eq.(1)$$

. Obtain the expression for the S-matrix element

$$S_{fi} = \delta_{fi} - ig\epsilon \int d^4y \bar{\psi}(y)\phi(y)\Psi(y)$$

where $\Psi(x)$ is the solution of Eq.(1) and can be written as

$$\Psi(x) = \psi(x) + g \int d^4y S_F(x-y)\phi(y)\Psi(y)$$

where $\psi(x)$ is the solution of a free particle Dirac equation.$\epsilon = (-1)^n$, where n= the number of antiparticles at time $t \to -\infty$.

34. Draw the Feynaman diagramms to order g^2 for the scattering (we will call the particle represented by the field ϕ as b and by the field ψ as f)

$$b(k_i) + f(p_i) \to b(k_f) + f(p_f)$$

.

and write the matrix element for the process.

35. Let the scalar field $\phi(x)$ represent a π^- meson. Introduce the electromagnetic interaction using the gauge principle and write down the Klein Gordon eqaution in the presence of an electromagnetic vector potential A_μ. Use this to wrie an expression for J.