Chennai Mathematical Institute

Ph.D.(Physics) IInd Semester Jan-Apr 2020

Quantum Field Theory | Comprehension Check | Jan 30, 2020

In classical theory *cross section* ([1]) is easily computed by solving the equations of motion. The soluton gives the equation of orbit which in turn gives *scattering angle* in terms of parameters of the incident beam. For scattering of particle from a *spherically symmetric potential* $V(r)$, the scattering angle can be written in terms of two constants of motion ([2]) . Expressing *impact parameter* b in terms of the scattering angle θ impact parameter (**??**) b, the differential cross section ([3]) is easily computed from the formula

$$\frac{d\sigma}{d\theta} = \frac{b}{\sin\theta}\frac{db(\theta)}{d\theta}. \tag{1}$$

In qunatum theory, however, this approach cannot work because the *trajectories are not well defined* ([4]) . All information must come from the wave function. So how do we formulate the scattering problem and extract cross section from the wavefunction?

There are two approaches to scattering cross section in quantum theory. The first one is a time independent approach and the second one is time dependent approach.

In the time dependent approach one computes a transition probability per unit time which gets related to the differential cross section. Basically one asks a question what the probability that a wave packet travelling along the incident direction in distant past will be travelling in a specified direction after scattering. For this the time dependent Schrodinger equation has to be solved.

From here we compute the transtion probability per unit time ([10]) which gives the (differential cross section) cross section. Actually, one needs to sum the transition probability per unit time to a group of final states.

Most text books also give a time independent approach to scattering problem in qunatum mechanics ([8]) . The computation of cross section uses an interpretation of wave function ([11]) . In this approach one needsto solve the time independent Schrodinger equation and obtain solution subject to a boundary condition speficfic to the scattering problem. ([10]) .

$$c_{\rm fi}(t) = \frac{1}{i\hbar}\int_0^t \langle f|H_I|i\rangle\, dt. \tag{2}$$

Answer the following questions in brief.

[1] Define all terms types in italics.

[2] What are these two constants of motion?

[3] Explain impact parameter by means of a diagram.

[4] Briefly explain the formula (1) for cross section.

[5] Why one says that the trajectories are not well defined in quantum theory.

[6] Why one needs to compute the probbility per unit time and not probabilty?

[7] When we do not need to sum over a group of final states? Explain with the help of an example.

[8] How can a time independent approach be used to desscribe scattering process which is esssentially a time dependent phenomenon?

[9] Explain the interpretation of wave function that is needed in potential scattering?

[10] How can one formulate the cross section calculationin terms of some trnasition probability per unit time?

[11] Give the boundary condition that is imposed on the solutiions of the time independent Schrodinger equation.

[12] The cross section has dimension of area? Explain what is this area associated with cross section?

Due Date::Feb 3, 2020