Chennai Mathematical Institute

Ph.D.(Physics) IInd Semester Jan-Apr 2020

Quantum Field Theory | Home Work-I | Jan 25, 2020

[1] Let $|n\rangle$ denote the n^{th} excited state of a harmonic oscillator. Show that [2]

$$\langle n|x|m\rangle = \sqrt{\frac{\hbar}{2m\omega}}\Big(\sqrt{n+1}\delta_{m,n+1} + \sqrt{n}\delta_{m,n-1}\Big) \tag{1}$$

[2] Using the properties of the ladder operators $a, a^\dagger$ and the number operator N, compute the average values of kinetic and potential energies for a harmonic oscillator in the n^{th} state $|n\rangle$. Verify that their sum equals $(n+1/2)\hbar\omega$. [4]

[3] Rearrange the operator expression $pqpq^2$ as sum of expressions of the form $\sum_{m,n} c_{mn} q_n p_m$ in which each term has all q operators on the left and all p operators on the right.

[4] Give a short argument that the matrix elements $\langle n|x^2|m\rangle$ and $\langle n|p^2|m\rangle$, are zero unless m has one of the values $n, n-2, n+2$. [2]

[5] A normal product of operators $a, a^\dagger$ is an expression having all raising operators on the left and all lowering operators on the right. As an example, $aa^\dagger$ can be written as $a^\dagger a + 1$, where the operators appear in the normal product form. What is the expectation value of such a product in the state $|0\rangle$? Express the following operators as a sum of operators in the normal product form and a constant. [4]

(*a*) $aa^{\dagger 2}a^2a^{\dagger 2}$, (*b*) $a^{\dagger 2}aa^{\dagger 2}a$.

[6] What is the value of zero point energy of a single harmonic oscillator? Schrodinger field is a collection of harmonic oscillators. What do you expect to the answer for zero point energy (minimum energy) Schrodinger field? Justify your answer. [4]

Due Date::Feb 3, 2020