University of Hyderabad

School of Physics

Jul 2010 - Dec 2010 | Time : 1hr
M.Sc. III-Semester | Quantum Mechanics-II | MM : 20

Session VI::Tutorial   Integral Equation

[1] Verify that the Green function

$$G_E(x) = -i\frac{m}{\hbar^2 k}\exp(ik|x|)$$

where $k = \sqrt{2mE/\hbar^2}$ satisfies the one dimensional equation

$$\left(\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + E\right) G_E(x) = \delta(x) \qquad (1)$$

[2] Using the Green function given in Q[1]

(a) Set up the integral equation for reflection and transmission coefficients for the one dimensional problems.

(b) Verify that the Green function gives the correct boundary conditions for the wave function at infinity.

(c) Solve the integral equation exactly for $V(x) = -\gamma\delta(x)$ and find the reflection coefficient.

[3] (a) Use the Green function as in Q[1] and find an approximate expression for reflection coefficient for an arbitrary potential $V(x)$.

(b) Show that for a square well potential

$$V(x) = \begin{cases} -V_0 & \text{for } |x| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

approximate reflection coefficient is given by

$$R \approx \frac{m^2 V_0^2}{\hbar^4 k^4}\sin^2 ka$$

Note:

The derivative of θ- function is Dirac δ- function:

$$\frac{d\theta(x)}{dx} = \delta(x)$$

The above result is a special case of the result that the derivative of a function discontinuous at a point of discontinuity $x0$ is given by

$$\frac{df(x)}{dx} = (b - a)\delta(x)$$

where a and b are the values of $f(x)$ as x approaches x_0 from the left and from the right repectively. For more on Dirac delta function see a recent write up on moodle course site.

Date : Aug 16,2010