University of Hyderabad

School of Physics

Jan 2010 - Apr 2010 | Time : 1hr
M.Sc. II-Semester | Quantum Mechanics-I | MM : 20

Tutorial-VII : Potential Problems in One Dimension

⊘ In the following problems V_0 is positive.

[1] For a particle in one dimension moving in the potential step Fig.1

$$V(x) = \begin{cases} 0 & x \le 0 \\ V_0 & x > 0 \end{cases} \tag{1}$$

(a) Find the energy eigen-functions for $E < V_0$. Verify that the energy eigenvalues are non-degenerate and continuous in this case.

(b) Show that the energy eigenvalues are doubly degenrate for $E > V_0$. Find two linearly independent solutions u_1 and u_2 for energy E such that the energy eigenfunctions are a linear combination of the two solutions.

[2] Solve the Schrodinger equation for the square well with a rigid wall, see Fig.2

$$V(x) = \begin{cases} \infty, & x \le 0 \\ 0, & 0 \le x \le L \\ V_0, & x > L \end{cases} \tag{2}$$

Find the energy eigenfunctions and

(a) derive the energy eigen-value condition for the bound states.

(b) verify that E is continuous and non-degenerate in the range $E > V_0$

[3] Find the eigenfucntions for the bound states for the potential

$$V(x) = \begin{cases} V_1, & x < a \\ V_2, & a < x < b \\ V_3, & x > b \end{cases}$$

($V_2 < V_1 < V_3$) and derive the energy quantization condition

$$\tan^{-1}(k/\alpha) + \tan^{-1}(k/\beta) = n\pi - k(b-a)$$

where

$$k = \sqrt{2m(E - V_2)/\hbar^2}, \qquad \alpha = \sqrt{2m(V_1 - E)/\hbar^2}, \qquad \beta = \sqrt{2m(V_1 - E)/\hbar^2}$$

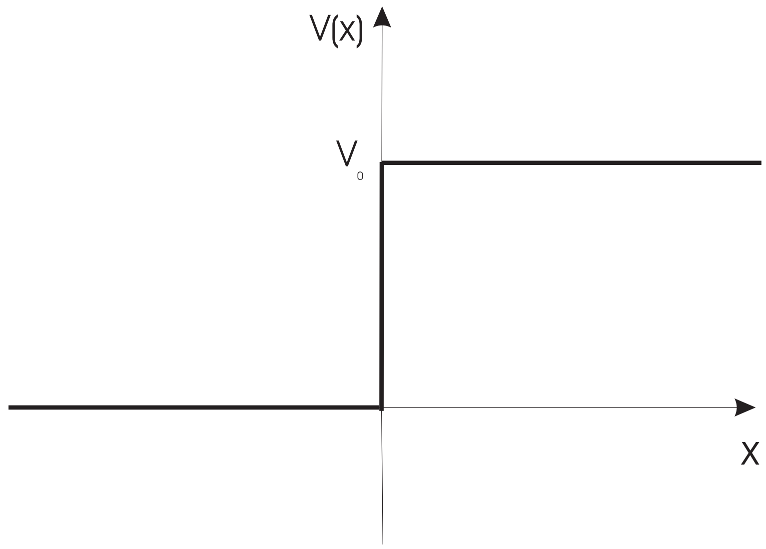

Fig. 1 for Q[1] : Potential Step

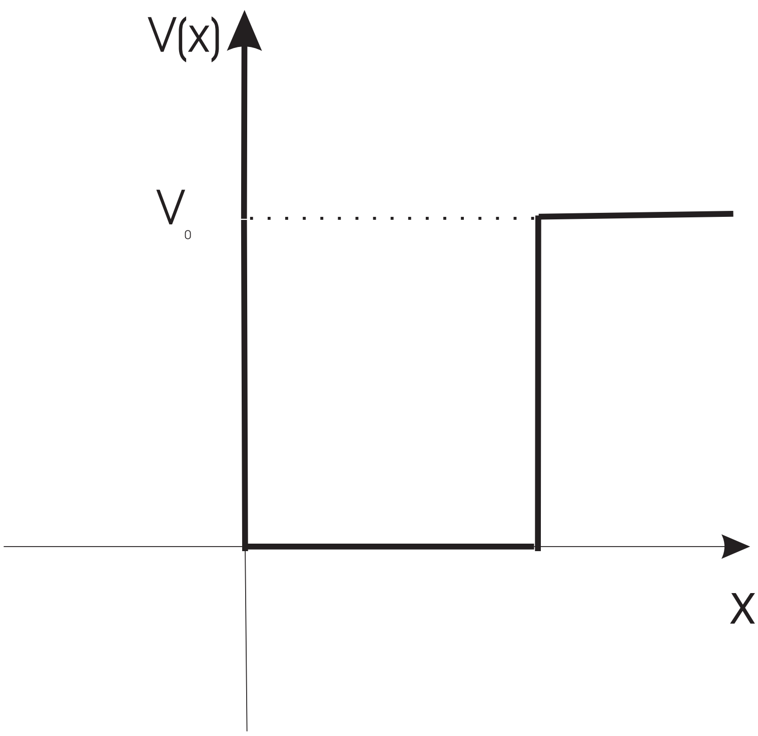

Fig. 2 for Q[2] : Well with a Wall