University of Hyderabad

School of Physics

Jan 2010 - Apr 2010 | Time : 1hr
M.Sc. II-Semester | Quantum Mechanics-I | MM : 20

Tutorial-I : Postulates of Quantum Mechanics

⊘ The vector space needed to describe a particular physical system is two dimensional complex vector space $\mathbb{C}^2$. The states are, therefore, represented by 2 component complex column vectors. The observables for this system are 2×2 matrices. A set of three dynamical variables of the system, X, Y, Z, are be represented by 2×2 matrices denoted by $\sigma_x, \sigma_y, \sigma_z$, where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

[1] Group the following vectors into sets such that each member in a set represents the same state.

$$\chi_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}; \chi_2 = \begin{pmatrix} 1+i \\ 1-i \end{pmatrix}; \chi_3 = \begin{pmatrix} -1+i \\ 1+i \end{pmatrix}; \chi_4 = \begin{pmatrix} 1 \\ -i \end{pmatrix}; \chi_5 = \begin{pmatrix} i \\ -1 \end{pmatrix}$$

[2] State which of the following operators can represent an observable quantity and which ones cannot represent an observable.

(a) $X_P = X + iY$ (b) $X_M = X - iY$

(c) $R = 3X + 12Y + 4Z$ (d) $T = X^2 + Y^2 + Z^2$

[3] What values are experimentally allowed if one measures the dynamical variable

(a) X (b) Y (c) Z (d) $T = X^2 + Y^2 + Z^2$

[4] Which vector would represent the state of the system, if it is known that the system has a definite value $+1$ for the dynamical variable X? What vector would represent the state if the system has a definite value -1 for the variable Y.

[5] Show that the allowed values of

$$X_{\hat{n}} = n_1 X + n_2 Y + n_3 Z$$

are given by $\pm\sqrt{n_1^2 + n_2^2 + n_3^2}$.

[6] If the state vector of the system is given by

$$\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} \end{pmatrix}$$

Find the probability that

(a) a measurement of X will give value 1
(b) a measurement of Y will give value -1.
(c) a measurement of Z will give value -1.
(d) a measurement of $X + Z$ will give value $\sqrt{2}$.