University of Hyderabad

School of Physics

Jan 2010 - Apr 2010 | Time : 1hr
M.Sc. II-Semester | Quantum Mechanics-I | MM : 20

Tutorial-V : Matrix Elements Using Algebraic Methods

⊘ In this set $|n\rangle$ represents the n^{th} energy eigenvector for the harmonic oscillator.

[1] Rearrange the operator expression $pqpq^2$ as sum of expressions of the form $umc_{mn}q^n p^m$ in which each term has all q operators on the left and all p operaotrs on the right.

[2] A normal product of operators $a, a^\dagger$ is an expression having all raising operators on the left and all lowering operators on the right. As an example, $aa^\dagger$ can be written as $a^\dagger a + 1$, where the operators appear in the normal product form. What is the expectation value of such a product in the state †0 ? Express the following operators as a sum of operators in the normal product form and a constant.

(a) $aa^{\dagger 2}aa\dagger^2$,

(b) $a^{\dagger 2}aa^\dagger a$.

[3] Using the properties of the ladder operators $a, a^\dagger$ and the number operator N, compute the average values of kinetic and potential energies for a harmonic oscillator in the nth state $|n\rangle$. Verify that the sum equals $(n + 1/2)\hbar\omega$.

[4] Give a short argument that the matrix elements $\langle m|x^2|n\rangle$ and $\langle m|p^2|n\rangle$, are zero unless m has one of the values $n, n-2, n+2$.

[5] Compute the average value $\langle n|x^4|n\rangle$.