University of Hyderabad

School of Physics

Jan 2010 - Apr 2010 | Time : 1hr
M.Sc. II-Semester | Quantum Mechanics-I | MM : 20

Tutorial-II : Calculating Commutators

Answer the following questions assuming the canonical commutation relations between the position and the momentum operators. For one degree of freedom

$$[q, p] = i\hbar;$$

For several degrees of freedom, the non-zero commutators are

$$[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$$

and all other commutators such as $[x, y], [x, p_y]$ etc. are all zero.

[1] Prove that

$$[q, p^N] = iN\hbar p^{N-1}$$
$$[p, q^N] = -iN\hbar q^{N-1}$$

[2] Let

$$a = \frac{1}{\sqrt{2m\omega\hbar}}(p - im\omega q)$$
$$a^\dagger = \frac{1}{\sqrt{2m\omega\hbar}}(p + im\omega q)$$

and $N = a^\dagger a$

(a) Compute the commutator $[a, a^\dagger]$

(b) Use results in (a) and find the commutators

$$[N, a] \text{ and } [N, a^\dagger]$$

(c) Express the harmonic oscillator hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$$

in terms of a and $a^\dagger$.

[3] Let

$$L_x = yp_z - zp_y, \quad L_y = zp_x - xp_z, \quad \text{and } L_z = xp_y - yp_x$$

be the angular momentum operators. Prove any one of the following.

$$[L_x, L_y] = i\hbar L_z$$

$$[L_y, L_z] = i\hbar L_x$$

$$[L_z, L_x] = i\hbar L_y$$

[4] Let $L\pm$ and L^2 be defined as

$$L_{\pm} = L_x \pm iL_y \text{ and } L^2 = L_x^2 + L_y^2 + L_z^2.$$

Prove that

(a) $[L_+, L_-] = 2\hbar L_z$

(b) $[L_z, L_\pm] = \pm\hbar L_\pm$

(c) $L_+L_- = L^2 - L_z^2 + \hbar L_z$

(d) $L^2 = \frac{1}{2}(L_+L_- + L_-L_+) + L_z^2$

Date : Aug 9, 2010