

Find rotation matrix for a rotation by an angle  $\alpha$  about the axis  $1, 2, 1$  where  $\cos \alpha = \frac{3}{5}, \sin \alpha = \frac{4}{5}$ .

**⊙ Solution:** The unit vector along the direction  $(1, 2, 1)$  is given by  $\hat{n} = \frac{1}{\sqrt{6}}(1, 2, 1)$ .

Under a rotation by an angle  $\alpha$  about axis  $\hat{n} = (n_1, n_2, n_3)$ , the new components  $\vec{X}$  are related to old components  $\vec{x}$  by equation

$$\vec{X} = \vec{x} - \sin \alpha (\hat{n} \times \vec{x}) + (1 - \cos \alpha) \hat{n} \times (\hat{n} \times \vec{x}) \quad (1)$$

We compute

$$\hat{n} \times \vec{x} = (n_2 x_3 - n_3 x_2, n_3 x_1 - n_1 x_3, n_1 x_2 - n_2 x_1)$$

$$(\hat{n} \times (\hat{n} \times \vec{x}))_1 = n_2(n_1 x_2 - n_2 x_1) - n_3(n_3 x_1 - n_1 x_3) \quad (2)$$

$$(\hat{n} \times (\hat{n} \times \vec{x}))_2 = n_3(n_2 x_3 - n_3 x_2) - n_1(n_1 x_2 - n_2 x_1) \quad (3)$$

$$(\hat{n} \times (\hat{n} \times \vec{x}))_3 = n_1(n_3 x_1 - n_1 x_3) - n_2(n_2 x_3 - n_3 x_2) \quad (4)$$

Therefore

$$X_1 = (1 - \cos \alpha)(n_2(n_1 x_2 - n_2 x_1) - n_3(n_3 x_1 - n_1 x_3)) - \sin \alpha(n_2 x_3 - n_3 x_2) + x_1$$

$$X_2 = (1 - \cos \alpha)(n_3(n_2 x_3 - n_3 x_2) - n_1(n_1 x_2 - n_2 x_1)) - \sin \alpha(n_3 x_1 - n_1 x_3) + x_2$$

$$X_3 = (1 - \cos \alpha)(n_1(n_3 x_1 - n_1 x_3) - n_2(n_2 x_3 - n_3 x_2)) - \sin \alpha(n_1 x_2 - n_2 x_1) + x_3$$

Therefore

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \mathbb{R} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \quad (5)$$

where the matrix  $\mathbb{R}$  is given by

$$\begin{pmatrix} -(1 - \cos \alpha)(n_2^2 + n_3^2) + 1 & (1 - \cos \alpha)n_1 n_2 + \sin \alpha n_3 & (1 - \cos \alpha)n_3 n_1 - \sin \alpha n_2 \\ (1 - \cos \alpha)n_1 n_2 - n_3 \sin \alpha & -(1 - \cos \alpha)(n_3^2 + n_1^2) + 1 & (1 - \cos \alpha)n_2 n_3 + n_1 \sin \alpha \\ (1 - \cos \alpha)n_1 n_3 + n_2 \sin \alpha & (1 - \cos \alpha)n_2 n_3 - n_1 \sin \alpha & -(1 - \cos \alpha)(n_1^2 + n_2^2) + 1 \end{pmatrix}$$

Substituting values

$$\hat{n} = \frac{1}{\sqrt{6}}(1, 2, 1), \quad (1 - \cos \alpha) = \frac{2}{5}, \quad \sin \alpha = \frac{4}{5}$$

and simplifying gives

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{15}(1 + \sqrt{6}) & \frac{1}{15}(1 - 4\sqrt{6}) \\ \frac{2}{15}(1 - \sqrt{6}) & \frac{13}{15} & \frac{2}{15}(1 + \sqrt{6}) \\ \frac{1}{15}(1 + 4\sqrt{6}) & \frac{2}{15}(1 - \sqrt{6}) & \frac{2}{3} \end{pmatrix} \quad (6)$$

PROOFS

LICENSE: CREATIVE COMMONS

NO WARRANTY, IMPLIED OR OTHERWISE

Open  File

View PDF

<http://ospace.org/node/3280>

Open from Ospace

---