(*a*) A satellite of mass 2000 kg is to be put into a circular orbit around the earth of radius 1100 km. What is the minimum energy required?

(*b*) What will the minimum energy required to transfer it to an elliptic orbit having minimum and maximum distances 1100km and 4100km?

☺ Solution:

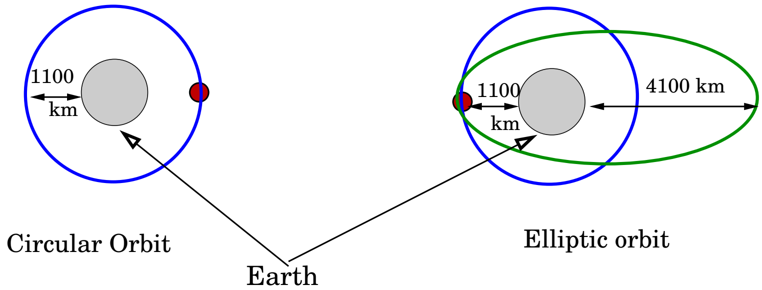

Fig. **1**

Initial data $R_e = 6400\text{km}$, $GM/R_e^2 = g \Rightarrow GM = gR_e^2$ Initial energy E_i of the satellite on the surface of the earth is

$$E_i = -\frac{GMm}{R_e} = -\frac{gR_e^2 m}{R_e} \tag{1}$$

$$= -gR_e^2 m = (9.8(\text{m/s}^2))(6400 \times 1000\,\text{m})(2000\,\text{kg}) \tag{2}$$

$$= -9.8 \times 128 \times 10^8 \tag{3}$$

$$= -12.5 \times 10^{10} J \tag{4}$$

Final energy of the satellite in the orbit at height of 1100 km be E_f.

$$E_f = -\frac{GM,}{2(R+h)} = -\frac{gR_e^2 m}{2(R_e+h)} \tag{5}$$

$$= -\frac{9.8 \times (6400 \times 6400 \times 10^6) \times 2000}{2 \times 7500 \times 10^3} \tag{6}$$

$$= \frac{9.8 \times 64 \times 64}{2 \times 75} \times 10^8 \tag{7}$$

$$= -5.35 \times 10^{10} \tag{8}$$

Therefore the energy required to put the satellite in the circular orbit at a height 1100km

$E_f - E_i = (-5.35 + !2.5) \times 10^{10}\text{J} = 7.15 \times 10^{10}\text{J}.$

Elliptic orbit Let r_1, r_2, a be, respectively, the perigee, apogee and the semi-major axis of the elliptic orbit. Then $2a = r_1 + r_2$ It is give that

$$r_1 = 1100km + R_e, r_2 = 4100km + R_e$$

Therefore

$$2a = 1100 + 4100 + 2 \times 6400 = 1800\,\text{km}$$

Hence a=9000km. The energy of the satellite in the elliptic orbit is

$$\begin{aligned}
E_{\text{ell}} &= -\frac{GMm}{2a} = -\frac{gR_e^2 m}{2a} && (9)\\
&= -\frac{9.8 \times 6400 \times 6400 \times 10^6 \times 2000}{1800 \times 10^3} && (10)\\
&= -\frac{9.8 \times 64^2 \times 2 \times 10^5}{18} && (11)\\
&= -4.4 \times 10^{10} J. && (12)
\end{aligned}$$

Therefore energy requires to transfer the satellite from the circular orbit to the elliptic orbit is $(-4.4 + 5.35) \times 10^{10}$ J $= 9.3 \times 10^9$J.