☹ me-que-12003

J.S.Plaskett's star is one of the most massive stars known at present. It is a double or a binary star, that is, it consists of two stars bound together by gravity. From spectroscopic studies it is known that the period of revolution of each component is 14.4 days; the velocity of each component is about 220 km/s; The orbit is nearly circular

(a) Argue that the masses of two stars are nearly equal and that they are nearly equidistant from the centre of mass of the system. [4]

(b) Compute the reduced mass and and the separation of the two components. [4]

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(*a*) Assume that the masses, velocities, radii of the circular orbits of the two stars are $m_{1,2}v_{1,2}, r_{1,2}$. Let F_1, F_2 be the gravitational pulls on a star due to the other star. Since the orbits are given to be circular

$$\frac{m_1 v_1^2}{r_1} = F_1, \qquad \frac{m_2 v_2^2}{r_2} = F_2, \tag{1}$$

Now use $F_! = F_2$ to get

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} \Longrightarrow \frac{m_1}{r_1} = \frac{m_2}{r_2} \tag{2}$$

the last step follows because it is given that $v_1 = v_2$. For a circular orbit, the speed is constant and time period is equal to (perimeter/ velocity):

$$T_1 = \frac{2\pi r_1}{v_1}, \qquad T_2 = \frac{2\pi r_2}{v_2} \tag{3}$$

We are given $T_1 = T_2$ and $v_1 = v_2$, hence we get $r_1 = r_2$, and Eq.(2) then then gives $m_1 = m_2$.

(*b*) The radius of the orbits can be obtained from (3):

$$\begin{aligned} r_1 &= \frac{T_1 v_1}{2\pi} && (4)\\ &= \frac{\boxed{14.4 \times 24 \times 60} \times 60 \ \text{sec})(220 \times 10^3 \text{m/s})}{2\pi}, \text{ use } \boxed{(14.4 \times 24) \times 60 \sim (14 \times (25 \times 60))} \\ &\approx (\boxed{14 \times 1500})(10)(220) \times 10^3 \quad \boxed{\text{used } (60/2\pi) \sim 10} && (5)\\ &\approx (200,00)(22) \times 10^5, \boxed{\text{Recall } 15^2 = 225}, \text{ so used } \boxed{(14)(15) \sim 200} && (6)\\ &\approx 4.4 \times 10^{10}\text{m}. && (7)\\ & && (8) \end{aligned}$$

Thus the distance between stars is $2r_1 \sim 8.8 \times 10^{10}$ m.

The mass can be computed by using the second law

$$\frac{mv^2}{r_1} = \frac{Gmm}{(2r_1)^2} \Rightarrow m = \frac{4v^2 r_1}{G} \tag{9}$$

Substituting numerical values

$$m = \frac{4 \times 220^2 \times 10^6 \times 4.4 \times 10^{10}}{6.67 \times 10^{-11}} = \frac{4 \times 484 \times 4.4 \times 10^{29}}{6.67} \tag{10}$$

$$= 4 \times 484 \times \left(\frac{2}{3}\right) \times 10^{29} = 4 \times (1.61 \times 2) \times 10^{31} \tag{11}$$

$$= 4(3.2) \times 10^{31} = 1.28 \times 10^{32} \text{ kg.} \tag{12}$$