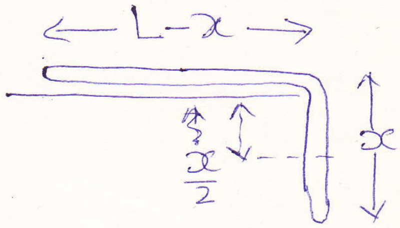
cm-que-01018

Variable mass problem

A rope of length l slides over the edge of a table. Initially a piece x_0 of it hangs without over the side of the table. Let x be the length of rope hanging vertically at time t. The rope is assumed to be perfectly flexible. Show that the principle of energy in the form $T+V=$ const. gives an integral of motion.

[Relative velocity of length added to the hanging mass is zero].



Let $x =$ length of the rope hanging over the edge.

$\rho =$ mass per unit length of the rope.

Since relative velocity of the mass added to hanging ~~ed~~ part is zero we can use Newton's Laws.

$$m\ddot{x} = \rho g x$$

$$\Rightarrow \quad l\ddot{x} = g x$$

Total energy $= \frac{1}{2}\rho l \dot{x}^2 - \frac{1}{2}\rho g x^2$

$$\frac{dE}{dt} = \rho l \dot{x}\ddot{x} - \rho g x \dot{x}$$
$$= \rho(l\ddot{x} - gx)\dot{x} = 0$$

Question Potential energy is written as $-\frac{1}{2}\rho g x$ explain why?

Question Derive EOM consider the two parts of the rope separately?

Variation