Variable mass problem

Falling chain: A chain lies pushed together at the edge of a table, except for a piece which hangs over it, initially at rest. The links of the chain start moving one at a time; neglect friction. The energy written in the usual form is no longer an integral of the motion. Instead, the impulsive (Carnot) energy loss must be taken into account in writing the balance of energy [Sommerfeld]

The relative velocity of link added is non zero and equal to – negative of the instantaneous velocity of the chain.

mass × acc of chain = external force + $\dot{m}$ × Vrel

Let x be the length of the chain hanging down at time t. ρ = mass per unit length

$$\rho x \ddot{x} = (\rho x) g x - (\rho \dot{x}) \dot{x}$$

or $$x\ddot{x} + \dot{x}^2 = gx$$