

Newtonian Mechanics

Motion on curves

$$x = R(\phi + \sin\phi)$$

$$y = R(1 - \cos\phi)$$

$$ds = \sqrt{\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2}$$

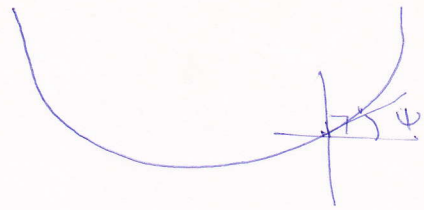
$$= \sqrt{R^2(1 + \cos\phi)^2 + R^2 \sin^2\phi}$$

$$= R\sqrt{1 + 2\cos\phi + \sin^2\phi + \cos^2\phi}$$

$$\frac{ds}{d\phi} = 2R \cos\phi/2$$

$$s = 4R \sin(\phi/2)$$

$$\psi = \frac{dx}{ds} \quad t = \left(\frac{dx}{ds}, \frac{dy}{ds}\right)$$



Tangential force = $-mg \sin\phi$
 $= -mg \frac{dy}{ds}$

EOM for ϕ

$$m \frac{d^2s}{dt^2} = -mg \frac{dy}{ds}$$

$$= -mg \frac{dy/ds}{d\phi/d\phi}$$

$$R \sin\phi / 4R \cos(\phi/2)^{1/2}$$

$$m \frac{d^2s}{dt^2} = -mg \cos(\phi/2)$$

$$= -mg R \sin(\phi/2)$$

$$= -mg \frac{s}{4R}$$

$$m \frac{d^2s}{dt^2} + \left(\frac{mg}{4R}\right) s = 0$$

$$\text{Frequency of oscill} = \sqrt{\frac{g}{4R}}$$