GRAVITATION AND COSMOLOGY: PRINCIPLES AND APPLICATIONS OF THE GENERAL THEORY OF RELATIVITY

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what we now call Riemannian geometry in his Göttingen inaugural lecture, *Über die Hypothesen, welche der Geometrie zu Grunde liegen*. Subsequent work by Christoffel, Ricci, Levi-Civita, Beltrami, and others developed Riemann's ideas into the beautiful mathematical structure described in our chapters on tensor analysis and curvature. However, it remained for Einstein to see the use physics could make of non-Euclidean geometry.

2 History of the Theory of Gravitation

At the end of the *Principia*, Isaac Newton (1642–1727) described gravitation as a cause that operates on the sun and planets "according to the quantity of solid matter which they contain and propagates on all sides to immense distances, decreasing always as the inverse square of the distances."[6] There are two parts to Newton's law, which were discovered in different ways, and which played different roles in the development of mechanics from Newton to Einstein.

It was of course Galileo Galilei (1564–1642) who discovered that bodies fall at a rate independent of their mass. His tools were an inclined plane to slow the fall, a water clock to measure its duration, and also a pendulum, to avoid rolling friction. These observations were later improved by Christaan Huygens (1629–1695). Newton could thus use his second law to conclude that the force exerted by gravitation is proportional to the mass of the body on which it acts; the third law then ensures that the force is also proportional to the mass of its source.

Newton was well aware that these conclusions might be only approximately true, and that the "inertial mass" entering in his second law might not be precisely the same as the "gravitational mass" appearing in the law of gravitation. If this were the case, we would have to write Newton's second law as

$$\mathbf{F} = m_i \mathbf{a} \tag{1.2.1}$$

and write the law of gravitation as

$$\mathbf{F} = m_g \mathbf{g} \tag{1.2.2}$$

where $\mathbf{g}$ is a field depending on position and other masses. The acceleration at a given point would be

$$\mathbf{a} = \left(\frac{m_g}{m_i}\right) \mathbf{g} \tag{1.2.3}$$

and would be different for bodies with different values for the ratio m_g/m_i; in particular pendulums of equal length would have periods proportional to $(m_i/m_g)^{1/2}$. Newton tested this possibility by experiments with pendulums of equal length but different composition, and found no difference in their periods. This result was later verified more accurately by Friedrich Wilhelm Bessel (1784–1846) in 1830. Then, in 1889, Roland von Eötvös[7] succeeded by a different method in showing that the ratio m_g/m_i does not differ from one substance to another by more than one part in 10^9. (See Figure 1.2.) Eötvös hung two weights A and B from the ends of a 40-cm beam suspended on a fine wire at its center. At equilibrium the beam would sag in such a way that

$$l_A(m_{gA}g - m_{iA}g_z') = l_B(m_{gB}g - m_{iB}g_z') \tag{1.2.4}$$

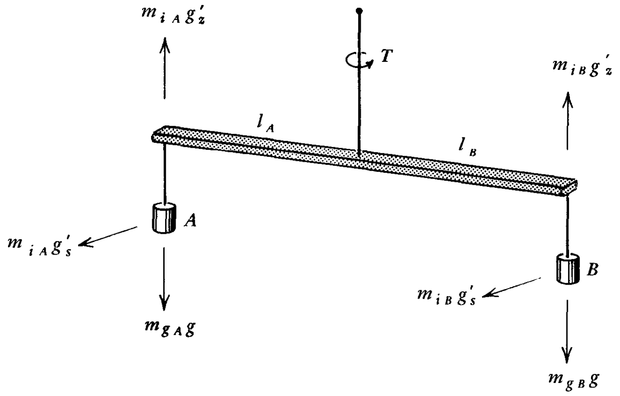

Figure 1.2 Schematic view of the Eötvös experiment.

where g is the earth's gravitational field, g_z' is the vertical component of the centripetal acceleration due to the earth's rotation, and l_A and l_B are the effective lever arms for the two weights. [Of course Eötvös chose weights and lever arms to be nearly equal, but the point of his method is that even if A is a little bigger than B, the beam will still sag just so as to make (1.2.4) correct.] At the latitude of Budapest the centripetal acceleration due to the earth's rotation also has an appreciable horizontal component g_s', giving to the balance a torque around the vertical axis equal to

$$T = l_A m_{iA} g_s' - l_B m_{iB} g_s'$$

Using the equilibrium condition to determine l_B, we have then

$$T = l_A m_{iA} g_s' \left[1 - \left(\frac{m_{gA}}{m_{iA}} g - g_z' \right) \left(\frac{m_{gB}}{m_{iB}} g - g_z' \right)^{-1} \right]$$

or, since g_z' is much less than g,

$$T = l_A g_s' m_{gA} \left[\frac{m_{iA}}{m_{gA}} - \frac{m_{iB}}{m_{gB}} \right]$$

Any inequality in the ratios m_i/m_g for the two weights would thus tend to twist the wire from which the balance was suspended. No twist was detected, and

Eötvös concluded from this that the difference of m_i/m_g for wood and platinum was less than 10^{-9}.

Einstein was very impressed with the observed equality of gravitational and inertial mass[8], and as we shall see, it served him as a signpost toward the Principle of Equivalence. (It also sets very stringent limits on any possible nongravitational forces that might exist. For instance, any new kind of electrostatic force in which the number of nucleons plays the role of charge would have to be much weaker than gravitation.[9]) In recent years a group under R. H. Dicke[10] at Princeton has improved on Eötvös' method, by using the gravitational field of the sun and the earth's centripetal acceleration toward the sun, rather than the rotation of the earth, to produce the torque on the balance. The advantage is that the angle between the direction of the sun and the balance arm changed with a 24-hr period, and so Dicke could filter out of his data any noise not at the diurnal frequency. In this way he concluded that "aluminum and gold fall toward the sun with the same acceleration, the accelerations differing from each other by at most one part in $10.^{11}$" It has also been shown (with very much less precision) that neutrons fall with the same acceleration as ordinary matter,[11] and that the gravitational force on electrons in copper is the same as on free electrons.[12]

We now move on to the second part of Newton's law of gravitation, which says that the force decreases as the inverse square of the distance. This idea was not entirely original with Newton. Johannus Scotus Erigena (c. 800–c. 877) had guessed that heaviness and lightness vary with distance from the earth. This theory was taken up by Adelard of Bath (twelfth century), who realized that a stone dropped into a very deep well could fall no farther than the center of the earth. (Incidentally, Adelard also translated Euclid from Arabic into Latin, thus making it available to medieval Europe.) The first suggestion of an inverse-square law may have been made around 1640 by Ismael Bullialdus (1605–1694). However, it was certainly Newton who in 1665 or 1666 first deduced the inverse-square law from observations. He knew that the moon falls toward the earth a distance 0.0045 ft. each second, and he knew that the moon is 60 earth radii away from the center of the earth. Hence, if the gravitational force obeys an inverse-square law, then an apple in Lincolnshire (which is 1 earth radius away from the center of the earth) should fall in the first second 3600 times 0.0045 ft, or about 16 ft, in good agreement with the measured value. However, Newton did not publish this calculation for twenty years, because he did not know how to justify the fact that he had treated the earth as if its whole mass were concentrated at its center. Meanwhile, it became known to several members of the Royal Society, including Edmund Halley (1656–1742), Christopher Wren (1632–1723), and Robert Hooke (1635–1703), that Kepler's third law would imply an inverse-square law of force if the orbits of planets were circular. That is, if the squares of the periods, r^2/v^2, are proportional to the cubes of the radii r^3, then the centripetal acceleration v^2/r is proportional to $1/r^2$. However, the planets actually move on ellipses, not circles, and no one knew how to calculate their centripetal acceleration. Under Halley's instigation, Newton in 1684 proved that planets moving under the influence of an inverse-square-law force would indeed obey all the empirical laws of Johannes Kepler (1571–1630); that is, they would move on ellipses with the sun at a focus, they would sweep out equal areas in equal times, and the square of their periods would be proportional to the cube of their major axes. Finally, in 1685, Newton was able to complete his lunar calculation of 1665. These stupendous accomplishments were published on July 5, 1686, under the title *Philosophiae Naturalis Principia Mathematica.*[13]

In the following centuries Newton's law of gravitation met with a brilliant series of successes in explaining the motion of the moon and planets. Some irregularities in the orbit of Uranus remained unexplained until, in 1846, they were independently used by John Couch Adams (1819–1892) in England and Urbain Jean Joseph LeVerrier (1811–1877) in France to predict the existence and position of Neptune. The discovery of Neptune shortly thereafter was perhaps the most splendid verification of Newton's theory. The motion of the moon and Encke's comet (and, later, Halley's comet) still showed departures from Newtonian theory, but it was clear that nongravitational forces could be at work.

One problem remained. A year before his prediction of Neptune, LeVerrier had calculated that the observed precession of the perihelia of Mercury was 35″/century faster than what would be expected according to Newton's theory from the known perturbing fields of the other planets. This discrepancy was confirmed in 1882 by Simon Newcomb (1835–1909), who gave a value of 43″ for the excess centennial precession.[14] LeVerrier had thought that this excess was probably due to a group of small planets between Mercury and the sun, but after a careful search none were discovered. Newcomb then suggested that perhaps the matter responsible for the faint "zodiacal light" seen in the plane of the ecliptic of the solar system was also responsible for the excess precession of Mercury. However, his calculations showed that the amount of matter needed to account for the precession of Mercury would, if placed in the plane of the ecliptic, produce a rotation of the plane of the orbits (that is, a precession of the nodes) of both Merqury and Venus different from what had been observed. For this reason, Newcomb was led by 1895 "to drop these explorations as unsatisfactory, and to prefer provisionally the hypothesis that the Sun's gravitation is not exactly as the inverse square."[15]

Unfortunately this was not the last word. In 1896 H. H. Seeliger constructed an elaborate model of the zodiacal light, placing the matter responsible on ellipsoids close to the sun, which could account for the excess precession of Mercury without upsetting the agreement between theory and experiment for the rotation of the planes of the inner planets' orbits. Today we know that this model is totally wrong, and that there simply is not enough interplanetary matter to account for the observed excess precession of Mercury. However, Seeliger's hypothesis, together with the continued success of Newtonian theory elsewhere, convinced Newcomb that there was no need to alter the law of gravitation.[15]

I do not know whether Einstein was very much influenced, in creating general relativity, by the problem of the precession of Mercury's perihelia. However, there is no doubt that the first confirmation of his theory was that it predicted an excess precession of precisely 43″/century.

3 History of the Principle of Relativity

Newtonian mechanics defined a family of reference frames, the so-called *inertial frames*, within which the laws of nature take the form given in the *Principia*. For instance, the equations for a system of point particles interacting gravitationally are

$$m_N \frac{d^2\mathbf{x}_N}{dt^2} = G \sum_M \frac{m_N m_M(\mathbf{x}_M - \mathbf{x}_N)}{|\mathbf{x}_M - \mathbf{x}_N|^3} \tag{1.3.1}$$

where m_N is the mass of the Nth particle and $\mathbf{x}_N$ is its Cartesian position vector at time t. It is a simple matter to check that these equations take the same form when written in terms of a new set of space-time coordinates:

$$\begin{aligned} \mathbf{x}' &= R\mathbf{x} + \mathbf{v}t + \mathbf{d} \\ t' &= t + \tau \end{aligned} \tag{1.3.2}$$

where $\mathbf{v}$, $\mathbf{d}$, and τ are any real constants, and R is any real orthogonal matrix. (If O and O′ use the unprimed and primed coordinate system, respectively, then O′ sees the O coordinate axes rotated by R, moving with velocity $\mathbf{v}$, displaced at $t = 0$ by $\mathbf{d}$, and O′ sees the O clock running behind his own by a time τ.) The transformations (1.3.2) form a 10-parameter group (three Euler angles in R, plus three components each for $\mathbf{v}$ and $\mathbf{d}$, plus one τ) today called the *Galileo group*, and the invariance of the laws of motion under such transformations is today called Galilean invariance, or the *Principle of Galilean Relativity*.

What really impressed Newton about all this was that there are a great many more transformations that do *not* leave the equations of motion invariant. For instance, (1.3.1) does not retain its form if we transform into an accelerating or a rotating coordinate system, that is, if we let $\mathbf{v}$ or R depend on t. The equations of motion can hold in their usual form in only a limited class of coordinate systems, called *inertial frames*. What then determines which reference frames are inertial frames? Newton answered that there must exist an absolute space, and that the inertial frames were those at rest in absolute space, or in a state of uniform motion with respect to absolute space. In his words[16].

> "Absolute space, in its own nature and with regard to anything external, always remains similar and unmovable. Relative space is some movable dimension or measure of absolute space, which our senses determine by its position with respect to other bodies, and is commonly taken for absolute space."

Newton also described several experiments that demonstrated what he interpreted as the effects of rotation with respect to absolute space. The most famous is the rotating bucket[17]:

> "If a bucket, suspended by a long cord, is so often turned about that finally the cord is strongly twisted, then is filled with water, and held at rest together with the water; and afterwards by the action of a second force, it is suddenly set whirling about the contrary way, and continues, while the cord is untwisting itself, for some time in this motion; the surface of the water will at first be level, just as it was before the vessel began to move; but subsequently the vessel, by gradually communicating its motion to the water, will make it begin sensibly to rotate, and the water will recede little by little from the middle and rise up at the sides of the vessel; its surface assuming a concave form. (This experiment I have made myself.) . . . At first, when the *relative* motion of the water in the vessel was greatest, that motion produced no tendency whatever of recession from the axis, the water made no endeavor to move upwards towards the circumference, by rising at the sides of the vessel, but remained level, and for that reason its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the rising of the water at the sides of the vessel indicated an endeavor to recede from the axis; and this endeavor reveals the real circular motion of the water, continually increasing till it had reached its greatest point, when relatively the water was at rest in the vessel. . . ."

Newton's conception of absolute space was rejected by his great opponent Gottfried Wilhelm von Leibniz (1646–1716), who argued that there is no philosophical need for any conception of space apart from the relations of material objects. The issue was debated in a famous series of letters[18] (1715–1716) between Leibniz and Newton's supporter, Samuel Clarke (1675–1729), and philosophers continued the argument, with Newton's position defended by Leonhard Euler (1707–1783) and Immanuel Kant (1724–1804) and attacked by Bishop George Berkeley (1685–1753) in his *Principles of Human Knowledge* (1710) and *Analyst* (1734). Of course none of this high-minded metaphysics led to any idea about how to develop a dynamical theory that might replace Newton's.

The first constructive attack on Newtonian absolute space was launched in the 1880's by the Austrian philosopher Ernst Mach (1836–1916). In his book *Die Mechanik in ihrer Entwicklung*[19] he remarks that

> "Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces *are* produced by its relative motion with respect to the mass of the Earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass until they were several leagues thick."

The hypothesis, that there is some influence of the mass of the Earth and the other celestial bodies" which determines the inertial frames, is called *Mach's principle.*

There is a simple experiment that anyone can perform on a starry night, to clarify the issues raised by Mach's principle. First stand still, and let your arms hang loose at your sides. Observe that the stars are more or less unmoving, and that your arms hang more or less straight down. Then pirouette. The stars will seem to rotate around the zenith, and at the same time your arms will be drawn upward by centrifugal force. It would surely be a remarkable coincidence if the inertial frame, in which your arms hung freely, just happened to be the reference frame in which typical stars are at rest, unless there were some interaction between the stars and you that determined your inertial frame.

This argument can be made more precise. The surface of the earth is not exactly an inertial frame, and of course the rotation and revolution of the earth give the stars an apparent motion, but these effects can be eliminated by using the inertial frame defined by the solar system as a whole. In this inertial frame of reference the average observed rotation of the galaxies with respect to any axis through the sun is less than about 1 arc-sec/century![20]

We seem to be faced with an unavoidable choice: Either we admit that there is a Newtonian absolute space-time, which defines the inertial frames and with respect to which typical galaxies happen to be at rest, or we must believe with Mach that inertia is due to an interaction with the average mass of the universe. And if Mach is right, then the acceleration given a particle by a given force ought to depend not only on the presence of the fixed stars but also, very slightly, on the distribution of matter in the immediate vicinity of the particle. We shall see in Chapter 3 that Einstein's equivalence principle gives an answer to the problem of inertia that does not refer to a Newtonian absolute space and yet does not quite agree with the conclusions of Mach. The issue is not closed.

I have not yet mentioned special relativity because, despite its name, it really does not affect the antinomy between absolute and relative space. However, we shall have to formulate the equivalence principle in special-relativistic terms, so a detailed review of special relativity is presented in the next chapter; for the moment we only take a glance at its history.

The theory of electrodynamics presented in 1864 by James Clark Maxwell (1831–1879) clearly did not satisfy the principle of Galilean relativity. For one thing, Maxwell's equations predict that the speed of light in vacuum is a universal constant c, but if this is true in one coordinate system x^i, t, then it will not be true in the "moving" coordinate system x'^i, t' defined by the Galilean transformation (1.3.2). Maxwell himself thought that electromagnetic waves were carried by a medium,[21] the luminiferous ether, so that his equations would hold in only a limited class of Galilean inertial frames, that is, in those coordinate frames at rest with respect to the ether.

However, all attempts to measure the velocity of the earth with respect to the ether failed,[22] even though the earth has a velocity of 30 km/sec relative to the sun, and about 200 km/sec relative to the center of our galaxy. The most important experiment was that of Albert Abraham Michelson (1852–1931) and E. W. Morley,[23] which showed in 1887 that the velocity of light is the same, within 5 km/sec, for light traveling along the direction of the earth's orbital motion and transverse to it. The accuracy of this result has been recently improved to about 1 km/sec.[24]

The persistent failure of experimentalists to discover effects of the earth's motion through the ether led theorists, including George Francis Fitzgerald[25] (1851–1901), Hendrik Antoon Lorentz[26] (1853–1928), and Jules Henri Poincaré[27] (1854–1912) to suggest reasons why such "ether drift" effects should be in principle unobservable. (See Figure 1.3.) Poincaré in particular seems to have glimpsed the revolutionary implications that this would have for mechanics, and Whittaker[28] gives the credit for special relativity to Poincaré and Lorentz. Without entering this controversy,[29] it is safe to say that a comprehensive solution to the problems

CONSEIL DE PHYSIQUE SOLVAY
BRUXELLES 1911

Photo Couprie, Bruxelles

GOLDSCHMIDT PLANCK RUBENS LINDEMANN HASENOHRL
NERNST BRILLOUIN SOMMERFELD DE BROGLIE HOSTELET
SOLVAY KNUDSEN HERZEN JEANS RUTHERFORD
LORENTZ WARBURG WIEN EINSTEIN LANGEVIN
PERRIN Madame CURIE POINCARÉ KAMERLINGH ONNES

Figure 1.3 Founders of the Special Theory of Relativity, at the First Solvay Conference in 1911.

of relativity in electrodynamics and mechanics was first set out in detail in 1905 by Albert Einstein[30] (1879–1955).

Einstein proposed that the Galilean transformation (1.3.2) should be replaced with a different 10-parameter space-time transformation, called a *Lorentz transformation,* that does leave Maxwell's equations and the speed of light invariant. (It is not clear that Einstein was directly influenced by the Michelson-Morley experiment itself,[31] but he specifically refers to "the unsuccessful attempts to discover any motion of the earth relative to the 'light medium'" in his 1905 paper.[32]) The equations of Newtonian mechanics, such as Eq. (1.3.1), are not invariant under Lorentz transformations; therefore Einstein was led to modify the laws of motion so that they would be Lorentz-invariant. The new physics, consisting of Maxwell's electrodynamics and Einstein's mechanics, then satisfied a new principle of relativity, the Principle of Special Relativity, which says that all physical equations must be invariant under Lorentz transformations. These developments are discussed in detail in the next chapter.

The Lorentz group of transformations is not in any way larger than the Galileo group, and therefore the principle of relativity was not originated by the special theory of relativity, but rather *restored* by it. Before Maxwell, it might have been supposed that all of physics is invariant under the Galileo group. Maxwell's equations were not invariant under this group, and for half a century it appeared that only mechanics, not electrodynamics, obeys the principle of relativity. After Einstein, it was clear that the equations of both mechanics and electrodynamics are invariant, but with respect to Lorentz transformations, not Galileo transformations. The laws of physics in the form given them by Maxwell and Einstein could still only be true in a limited class of inertial reference frames, and the question of what determines these inertial frames was as mysterious after 1905 as in 1686.

It remained to construct a relativistic theory of gravitation. A crucial step toward this goal was taken in 1907, when Einstein introduced the Principle of Equivalence of Gravitation and Inertia,[33] and used it to calculate the red shift of light in a gravitational field. As we shall see in Chapter 3, this principle determines the effects of gravitation on arbitrary physical systems, but it does not determine the field equations for gravitation itself. Einstein tried to use the equivalence principle in 1911 to calculate the deflection of light in the sun's gravitational field,[34] but the structure of the field was not then correctly understood, and Einstein's answer was one-half the "correct" general-relativistic result, derived here in Chapter 8. A number of attempts were made in 1911–1912 by Einstein,[35] Abraham,[36] and Nordström[37] to construct relativistic field equations for a single scalar gravitational field, but Einstein soon became dissatisfied with all such theories, largely on aesthetic grounds. (The gravitational deflection of light by the sun had not yet been measured.) A collaboration with the mathematician Marcel Grossman led Einstein by 1913 to the view[38] that the gravitational field must be identified with the 10 components of the metric tensor of Riemannian space-time geometry. As discussed in Chapters 4 and 5, the Principle of Equivalence is incorporated into this formalism through the requirement that the physical equations be invariant under *general* coordinate transformations, not just Lorentz transformations, though I do not know to what extent this "General Principle of Relativity" took on in Einstein's mind a life of its own, apart from the Principle of Equivalence. During the next two years, Einstein presented to the Prussian Academy of Sciences a series of papers[39] in which he worked out the field equations for the metric tensor and calculated the gravitational deflection of light and the precession of the perihelia of Mercury. These magnificent achievements were finally summarized by Einstein in his 1916 paper,[1] titled "The Foundation of the General Theory of Relativity."

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