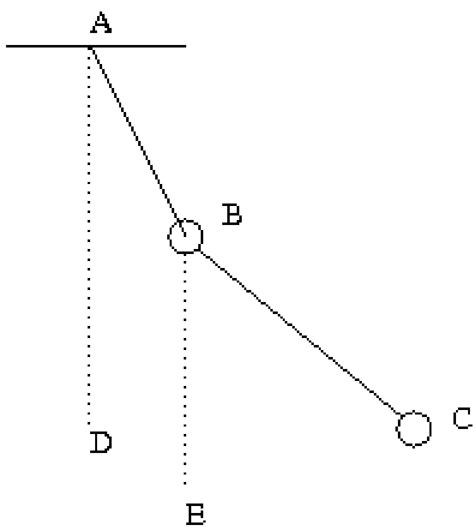
CHENNAI MATHEMATICAL INSTITUTE
CLASSICAL MECHANICS I
PROBLEM SHEET XI

5th Novmber 2012
date due 12th November 2012



51. Consider the double pendulum shown in the figure. Masses of the two bobs at B and C is m. AB = L_1 BC = L_2 the angle DAB = θ_1 and angle EDB = θ_2 , where AD and BE are vertical. Write the Lagrangian and the equations of motion for the angles θ_1 , θ_2. Write the Hamiltonian for the system.

52. Consider two equal masses A,B (M each) going around each other in a circular path of radius a about their centre of mass O with angular velocity $\omega = GM/(4a^3)$. A small satellite S of mass $m\,(\,m\; << \; M\,)$ is moving under the influence of the two body system in the same plane as the orbits of A and B. Let (r,θ) represent the polar coordinates of S in the rotating frame in which A and B are rest with O as origin. Write the Lagrangian for S in terms of (r,θ). Show that there are three points on the line joining AB (and extended) where the satellite is stationary (that is rotating along with the two body system. Find the points.

These points are known as Lagrange points L_1, L_2, L_3 - these are unstable. There are two more for which $\theta = 90^0\, 270^0$ - prove this-they are referred to as $L_4\, L_5$ and these are stable.Find the locations of L_4, L_5 (Stability analysis not required)

Note:There are similar points for the earth sun system (we ofcourse had to redo the anaysis with appropriate masses). In fact WMAP a satellite is located at L_2 , a point located approximately at $1.5\ \times 10^6 k.m.$ from the earth. This is a satellite which measures the cosmic microwave background anisotropy.

53. In the class I failed to make the point that one can commit a mistake if one is not careful about taking partial derivatives keeping in mind which variables are held constant. Now the following problem illustrates this.

Consider a free particle whose Lagrangian

$$L \;=\; T \;-\; V \;=\; T(q, \frac{dq}{dt})$$

We have the canonical momentum defined by

$$p \;=\; \frac{\partial L}{\partial \frac{dq}{dt}}$$

thus using

$$\frac{d}{dt}\frac{\partial L}{\partial \frac{dq}{dt}} \;=\; \frac{\partial L}{\partial q}$$

we get

$$\frac{dp}{dt} \;=\; \frac{\partial T}{\partial q} \tag{1}$$

Do a similar analysis for the Hamiltonian. $H \;=\; T \;+\; V \;=\; T$ and the Hamilton's equation of motion implies

$$\frac{dp}{dt} \;=\; -\frac{\partial H}{\partial q} = -\frac{\partial T}{\partial q} \tag{2}$$

Equations (1) and (2) seems to imply $\frac{dp}{dt} \;=\; 0$. However writing in terms of cylindrical coordinates (r, θ, z) the kinetic energy of a point particle of mass m is given by $T \;=\; \frac{1}{2}(m(\frac{dr}{dt})^2 \;+\; mr^2\,(\frac{d\phi}{dt})^2 \;+\; m(\frac{dz}{dt}^2)$ and $\frac{dp_r}{dt} \neq 0$. The problem is show this result in general. Consider a system with N degrees of freedom whose Lagrangian is

$$L(q_i, \frac{dq_i}{dt}, t) \;=\; \frac{1}{2}\sum_{ij}^{N} F_{ij}(q)\frac{dq_i}{dt}\frac{dq_j}{dt}$$

where $F_{ij} = F_{ji}$. and are functions of N-independent co-ordinated q_i.The canonical momenta are given by

$$p_i = F_{ij}\frac{dq_j}{dt}$$

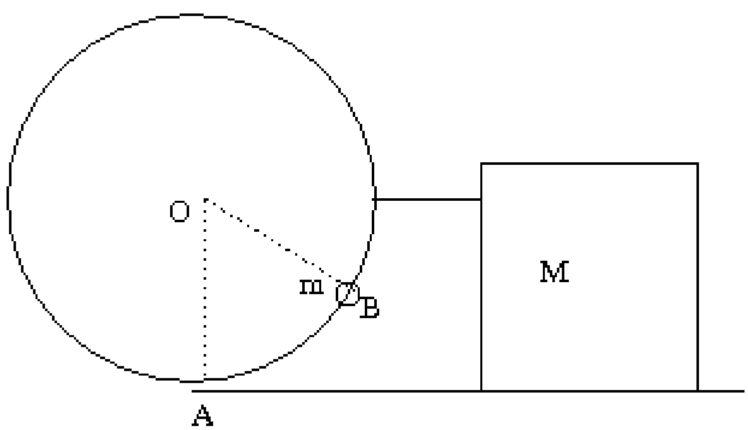
where the summation convention has been used. Let $F_{ij}^{-1}F_{jk} = \delta ik$. Show that

$$H(q_i, p_i, t) = \frac{1}{2}p_i(F^{-1})_{ij}p_j$$

Show that

$$\frac{\partial H(q,p,t)}{\partial q} = -\frac{\partial L(q, \frac{dq}{dt}, t)}{\partial q}$$

in spite of the fact $H = L = T$.



54. A block of mass M connected to massless ring of radius R by means of a massless rod. A bead of mass m is free to move slide on the ring. The motion is on a frictionless plane. Write the Lagrangian and the Hamiltonian of the system in terms of x the coordinate of M and the θ = angle AOB. Find the equations of motion. In the limit of small θ find θ as a function of time.

55. Consider the relativistic motion of particle of rest mass m under the action of a constant force along $\hat{\epsilon}_1$. If the particle starts at $t = 0$ from the origin of an inertial frame S , find $x_1(t)$ and show its speed never exceeds that of light.