CHENNAI MATHEMATICAL INSTITUTE
CLASSICAL MECHANICS I
PROBLEM SHEET VI

17th September 2012
date due 1st October 2012

Cassini, a space crarft, was lauched in 1997 and reached Saturn on June 2004 It picked up energy (Termed as Flyby effect, or gravity assist or Sling-shot effect) by passing near Venus (in 1998) and then Jupiter (in December 2000). Questions 26, 27, and 28 help you understand how this happens

Assume the planet is stationary during the time the space craft is near the planet. Further assume the space craft is only under the influence of the planet during its "flyby" - that is treat the velocity of the planet around the sun as a constant $\vec{V}$ during the time the space craft is near the planet. Let $\vec{V_i}, \vec{V_f}$ be the initial and the final velocities of the space craft as seen from the rest frame of the sun. Assume the mass of space craft is very small compared to the mass of the planet.

Further assume all the velocity vectors $(\vec{V}, \vec{V_i}, \vec{V_f})$ are in the same plane as the planet's orbit around the sun.

26. Let the angle the space craft "scatters' due to the "collision" with the planet be β (This is the angle between the initial and the final velocities as seen from the rest frame of the planet.)

Show that

$$V_f^2 \ = \ V_i^2 \ + \ 2(V^2(1-\cos(\beta)) \ + \ 2\mathrm{VV_i}[\cos(\alpha-\beta) - \cos(\alpha)]$$

where α is the angle between $\vec{V_i}$ and $\vec{V}$.

27. Let V_i, α and V be fixed. Show that the maximum of V_f occurs for

$$\tan(\beta_{\mathrm{m}}) \ = \frac{\mathrm{V_i}\sin(\alpha)}{\mathrm{V_i}\cos(\alpha) - \mathrm{V}}$$

Take $V_i \ = \ 1.5V, \alpha \ = 40^0$ and find the values of β_m . Treating V_f/V_i as a function of β,show that

$$\frac{V_f}{V_i}(cos(\beta_m + \theta)) \ = \ \frac{V_f}{V_i}(cos(\beta_m - \theta))$$

that is V_f/V_i as a function of β is symmetrical about β_m Find the allowed values of β for $V_f/V_i = 1.2$.

28. For Jupiter $V = 13$km/s, Mass $= 1.90 \times 10^{27}$kg. Find the closest approach of the satellite to Jupiter for the value of β's obtained in the previous problem. Given the radius of Jupiter is 7.14×10^4km. comment on your answer.

29. Given that the earth's satellite near the earth's surface and the satellite around the moon near the moon's surface take about the same time to make one revolution, what can you conclude about the moon's composition?

30 Derive the Rutherford scattering formula for the case of repulsive force.