☹ em-que-09005

A rod of mass m and length ℓ and resistance R starts from rest and slides on two parallel rails of zero resistance as shown in Figure. A uniform magnetic field fill the area A battery and is perpendicular and out of the plane of the paper. A battery of of voltage V is connected as shown in the figure.

[1] Describe what happens when the rod iskept on the rails at rest and released.

[2] Write down $F = m(\frac{dv}{dt})$ and integrate it so show that

$$v(t) = \frac{V}{B\ell}\Big(1 - \exp\Big(-\frac{B^2\ell^2 t}{mR}\Big)\Big).$$

Hint: Find the limiting speed and separate that out from the total v.

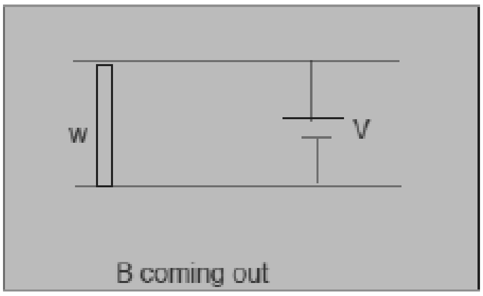

Fig. **1**

QAnalysis

Read the given question carefully, and think and have your answer(s)/comment(s) about the following points.

1. If there is no battery, in Figure below, will the rod move on its own? Will there be an induced current in the circuit?

2. Consider the same arrangement as in part (1) above and again there is no battery, but there is a magnetic field perpendicular in the circuit? Will there be an induced current? What if the magnetic field is decreasing with time? And what if, the magnetic field changes with time as $B_0 \sin\omega t$? If yes, what will be the effect of the induced current on the rod?

3. In the given problem, there is a DC source connected to the circuit. Will there be a magnetic field perpendicular to plane of the paper? Is the applied voltage changing with time? What will be its effect?

4. You are asked to argue that

   net EMF in the loops is $V = Bv\ell$ when the rod has speed v.

   It would then appear that there is an induced emf, $e = -BV\ell$, and the rod is moving.

5. In the given arrangement, can the rod move? WHY? Where will the force come from?
   If the rod does not move, can there be an induced emf in the circuit? When will there be an induced emf? When the rod is stationary, or when the rod is moving, or both?

6. Imagine that you have connected the battery, initially without the rod in its place. When the rod is put in its place, as shown in the figure of there will be a current at time $t = 0$. Compare the effect on the rod when (i) the battery is connected and (ii) when the battery is not connected?

7. Write all the forces on the rod when the battery is connected and set up the equation of motion.

8. There will be two opposing forces on the rod. Find the condition that these two forces are equal and add up zero.

9. The requirement in (7) gives a value of velocity. Is this terminal velocity? What do you understand by terminal velocity?

Serway and Jewett*

☺ **Solution:**

When the rod is kept on rails and released, a current flows because it is connected to a battery. If at any time a current I flows in the circuit, a magnetic force $BI\ell$ acts on the rod and it starts accelerating.

When the rod is moving, there is a change in net flux linked with the circuit giving rise to an emf $= \frac{d\phi}{dt}$. The rate of change of flux an be seen to be $Bv\ell$, as the rate of change of area is $v\ell$. This gives rise to an induced e.m.f.

$e = Bv\ell$. The current I at time t is given by $\mathcal{E}/R$ where $\mathcal{E}$ is the net e.m.f. in the circuit. Using Kirchoff's law we get

$$IR = V - B\ell v \tag{1}$$

The net force acting on the wire is

$$F = BI\ell = \frac{B\ell}{R}\Big(V - Bv\ell\Big). \tag{2}$$

Therefore, we get the equation of motion as

$$m\frac{dv}{dt} = \frac{B\ell}{R}\Big(V - Bv\ell\Big). \tag{3}$$

This equation can be integrated. To solve we we rewrite it as

$$\frac{dv}{V - B\ell v} = \frac{B\ell}{mR}. \tag{4}$$

Integrating we get

$$-\frac{1}{B\ell}\log(V - Bv\ell)\Big|_0^v = \frac{B\ell}{mR}\Big|_0^t \tag{5}$$

$$\text{or } \log\frac{(V - Bv\ell)}{V} = -\frac{B^2\ell^2}{mR}t \tag{6}$$

$$\text{or } V - Bv\ell = V\exp\Big(-\frac{B^2\ell^2 t}{mR}\Big). \tag{7}$$

Therefore, we have

$$v(t) = \frac{V}{B\ell}\Big\{1 - \exp\Big(-\frac{B^2\ell^2 t}{mR}\Big)\Big\} \tag{8}$$