☹ qm-que-06004

Show that the probability that electron in the ground state of hydrogen atom will be found outside the classical region is $13e^{-4}$. It is given that the normalized radial wave function of the electron, in the ground state, is

$$R_{10}(r) = 2\left(\frac{1}{a_0}\right)^{3/2} \exp(-r/a_0)$$

where $a_0 = \frac{\hbar^2}{me^2}$ is the Bohr radius and the energy levels of H atom are given to be $E_n = -\frac{e^2}{2a_0 n^2}$

☺*Solution*: The classical turning point(s) are obtained by equating $V(r) = E$ which the ground state gives $r = 2a_0$. Thus the classical region is $0 \le r \le 2a_0$. The probability that the electron will be found outside classical region is

$$\begin{aligned}
P(r > 2a_0) &= \int_{2a_0}^{\infty} |R_{10}(r)|^2)\, r^2 dr &(1)\\
&= \frac{1}{a_0^3}\int_{2a_0}^{\infty} 4\exp(-2r/a_0)\, dr &(2)\\
&= \frac{1}{2}\int_4^{\infty} e^{-t}t^2 dt, \qquad t = (2r/a_0) &(3)\\
&= \frac{1}{2}\Big(- e^{-t}t^2|_4^{\infty} + \int_4^{\infty} (2t)e^{-t}dt\Big) &(4)\\
&= \frac{1}{2}\Big(16e^{-4} - 2te^{-t}|_4^{\infty} + 2\int_4^{\infty} e^{-t}dt\Big) &(5)\\
&= \frac{1}{2}\Big(16e^{-4} + 8e^{-4} - 2e^{-t}|_4^{\infty}\Big) &(6)\\
&= \frac{1}{2}(16 + 8 + 2)e^{-4} &(7)\\
&= 13e^{-4}. &(8)
\end{aligned}$$