☹ em-que-02029

Find the direction and magnitude of $\vec{E}$ at the center of a rhombus, with interior angles of $\pi/3$ and $2\pi/3$, with charges at the corners as shown in figure below. Assume that $q = 1 \times 10^{-8}$C, $a = 5$cm

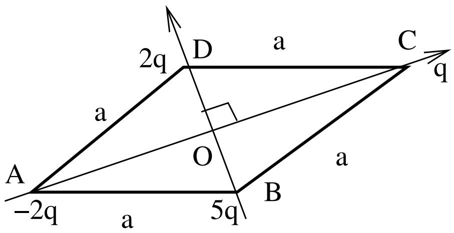

Fig. **1**

Solution: Note that for a rhombus the diagonals intersect at right angle. The interior angles are given to be $\pi/3$ and $2\pi/3$. If a is the side of rhombus, then the distances of the corners C, D from the center O are given by

$$d_1 = OC = a\cos(\pi/6) = \frac{\sqrt{3}}{2}2; \qquad d_2 = OD = \frac{a}{2} \tag{1}$$

Denote unit vectors along OC and OD as $\hat{u}, \hat{v}$ respectively the field at the center is given by

$$\begin{aligned} \vec{E} &= \frac{1}{4\pi\epsilon_0}\left(\frac{2q-q}{d_1^2}\hat{u} + \frac{-2q+5q}{d_2^2}\hat{v}\right) \\ &= \frac{q}{4\pi\epsilon_0}\left(\frac{4\hat{u}}{3a^2} + \frac{6\hat{v}}{a^2}\right) \end{aligned}$$

For $q = 1 \times 10^{-8}C, a = 5\,\text{cm}$ we get

$$\vec{E} = \frac{q}{4\pi\epsilon_0 a^2}(\frac{4}{3}\hat{u} + 6\hat{v}) =$$

To compute numerical values we use $\frac{1}{4\pi\epsilon_0} = \frac{1}{4\pi}(c^2/\mu_0) = c^2 \times 10^{-7} = 9 \times 10^9$. Therefore

$$\begin{aligned} \vec{E} &= \frac{q}{4\pi\epsilon_0 a^2}(\frac{4}{3}\hat{u} + 6\hat{v}) \\ &= \frac{10^{-8} \times 9 \times 10^9}{25 \times 10^{-4}}(\frac{4}{3}\hat{u} + 6\hat{v}) \\ &= 36 \times 10^3\text{V/m} \end{aligned} \tag{2}$$