$\boxed{\text{QFT-que-15002}}$

Decay rate for Λ hyperon

$$\Lambda^0 \longrightarrow p + \pi^- \qquad (1)$$

The interaction Hamiltonian density is

$$\mathcal{H}_{int} = \bar{\Psi}_p (g - g'\gamma_5) \Psi_\Lambda(x) \Phi_\pi^+ + h.c. \qquad (2)$$

$$S_{fi} = \delta_{fi} - i T_{fi}$$

$$T_{fi} = (2\pi)^4 \delta^4(Q - p - k) \sqrt{\frac{1}{2\omega_\pi}} \sqrt{\frac{M_\Lambda}{E_\Lambda}} \sqrt{\frac{M_p}{E_p}}\, m_{fi} \qquad (3)$$

We have

$$m_{fi} = \frac{\bar{u}^{(r)}(p)}{(2\pi)^{3/2}} (g - g'\gamma_5) \frac{u_\Lambda^{(s)}(Q)}{(2\pi)^{3/2}} \times \frac{1}{(2\pi)^{3/2}} \qquad (4)$$

Transition probability per unit time

$$= (2\pi)^4 \delta^{(4)}(Q - k - p) \frac{1}{2\omega_\pi} \left(\frac{E_p}{M_p}\right) \left(\frac{E_\Lambda}{M_\Lambda}\right) |m_{fi}|^2 \qquad (5)$$

We will work in the rest frame of the decaying particle.

$Q = (M_\Lambda, 0) \quad E_\Lambda = M_\Lambda \quad \vec{k} + \vec{p} = 0.$

The transition rate assumes the form.

$$= (2\pi)^4 \delta(M_\Lambda - E_p - E_\pi)\, \delta^{(3)}(\vec{k} + \vec{p}) \left(\frac{1}{2\omega_\pi}\right) \left(\frac{E_p}{M_p}\right) |m_{fi}|^2 \qquad (6)$$

$$\times (2\pi)^3$$

The $(2\pi)^3$-factor comes from 1/density for the decaying particle.

Here $1/\text{density} = (2\pi)^3 \dfrac{M_\Lambda}{E_\Lambda}$

The expression has to be summed over all final states because we are not observing momenta or spin values of the decay products.

Sum over final states

$$\rightarrow \int \frac{d^3k}{} \int \frac{d^3p}{} \quad (7)$$

We need to average (6) over all initial states. When Λ^0 particle is at rest it can exist in two spin states (spin of $\Lambda = 1/2$).

$\therefore$ aver over initial state $\Big\}$ $\frac{1}{(2s+1)} = \frac{1}{2}$ for Λ decay. (8)

Transition prob per unit time becomes.

$$\omega_{fi} = \iint (2\pi)^3 \times \left(\frac{1}{2}\right) \times (2\pi)^4 \, \delta^{(4)}(Q-p-k)\left(\frac{1}{2\omega}\right)\left(\frac{E_p}{M_p}\right)\left(\frac{M_p}{E_p}\right) |m_{fi}|^2 \frac{d^3k \; d^3p}{} \quad (9)$$

The integrals are done with the help of Dirac delta function. In the rest frame of Λ hyperon

$$\delta^{(4)}(Q-p-k) = \delta(M_\Lambda - E_p - \cancel{E_\pi}^{\,\omega_\pi})\,\delta(\vec{k}+\vec{p})$$

$$E_p = \sqrt{|\vec{p}|^2 + M_p^2} \qquad E_\pi = \sqrt{|\vec{k}|^2 + m_\pi^2} \quad (10)$$

Use $\delta^{(3)}(\vec{k}+\vec{p})$ to do $\vec{k}$ integrals. This means replacement $\vec{k} \to \vec{p}$ everywhere.

Next carry out d^3p integral using $\delta(M_\Lambda - E_p - \omega_\pi)$ for this purpose note that

$$d^3p = 4\pi |\vec{p}|^2 \, d|\vec{p}|$$

$$E_p = \sqrt{|\vec{p}|^2 + M_p^2}, \quad \omega_\pi = \sqrt{|\vec{p}|^2 + m_\pi^2} \qquad (11)$$

The presence of Dirac delta $\delta(M_\Lambda - E_p - \omega_\pi)$ means that $|\vec{p}|$ is to be computed using energy momentum conservation by solving

$$M_\Lambda - \sqrt{|\vec{p}|^2 + M_p^2} - \sqrt{|\vec{p}|^2 + m_\pi^2} = 0. \qquad (12)$$

If p_0 is a root of this equation then

$$\delta(M_\Lambda - E_p - \omega_\pi) = \frac{1}{|J|}\,\delta(|\vec{p}| - p_0) \qquad (13)$$

Where

$$J = \frac{d}{dp}\left(M_\Lambda - \sqrt{p^2 + M_p^2} - \sqrt{p^2 + m_\pi^2}\right)\Big|_{p=p_0}$$

$$= \left(-\frac{p}{\sqrt{p^2+M_p^2}} + \frac{p}{\sqrt{p^2+m_\pi^2}}\right)\Big|_{p=p_0}$$

$$= -2p\left(\frac{1}{E_p} + \frac{1}{\omega_\pi}\right)$$

$$= -2p\,\frac{(E_p + \omega_\pi)}{E_p\,\omega_\pi} = -\frac{2p_0 M_\Lambda}{E_p\,\omega_\pi} \qquad (14)$$

$$\mathcal{M}_{fi} = \bar{u}^{(r)}(p)\,(g+g'\gamma_5)\,u^{(s)}(Q) \quad \times (2\pi)^{-9/2}$$

$$|\mathcal{M}_{fi}|^2 = |\bar{u}^{(r)}(p)(g+g'\gamma_5)u^{(s)}(Q)|^2 \quad \times \frac{1}{(2\pi)^9}$$

$$= \bar{u}^{(s)}(Q)(g^* - g'^*\gamma_5)u^{(r)}(p)\,\bar{u}^{(r)}(p)(g+g'\gamma_5)u^{(s)}(Q)$$

$$= u^{(s)}(Q)\bar{u}^{(s)}(Q)(g^* - g'\gamma_5)\,u^{(r)}(p)\bar{u}^{(r)}(p)(g+g'\gamma_5)$$

$$\sum_r\sum_s |\mathcal{M}_{fi}|^2 = \mathrm{Tr}\left[\left(\frac{\not{Q}+M_\Lambda}{2M_\Lambda}\right)(g^*-g'^*\gamma_5)\left(\frac{\not{p}+M_p}{2M_p}\right)(g+g'\gamma_5)\right]$$

$$= \mathrm{Tr}\left[\left(\frac{\not{Q}+M_\Lambda}{2M_\Lambda}\right)\left(\frac{\not{p}+m_p}{2m_p}\right)\right]|g|^2$$

$$-\mathrm{Tr}\left[\left(\frac{\not{Q}+M_\Lambda}{2M_\Lambda}\right)\gamma_5\,\frac{(\not{p}+M_p)}{2M_p}\gamma_5\right]|g'|^2$$

$$= \mathrm{Tr}\left[\frac{\not{Q}\not{p}+M_pM_\Lambda}{4M_pM_\Lambda}\right]|g|^2 \qquad \times(--)$$

$$\mp \mathrm{Tr}\left[\frac{-\not{Q}\not{p}+M_pM_\Lambda}{4M_pM_\Lambda}\right]|g'|^2$$

$$= \left(\frac{4\,Q.p+4M_pM_\Lambda}{4M_pM_\Lambda}\right)|g|^2$$

$$\mp 4|g'|^2\left(\frac{4M_pM_\Lambda - 4Q.p}{4M_pM_\Lambda}\right)$$

$$Q.p = Q_0p_0 - \vec{Q}\cdot\vec{P} = M_\Lambda E_p \qquad \vec{Q}=0.$$

(5)

$$\therefore |m_{fi}|^2 = \frac{1}{(2\pi)^9}\left(|g|^2\,\frac{Q\cdot p + M_p M_\Lambda}{M_p M_\Lambda} - |g'|^2\left(\frac{M_p M_\Lambda - Q\cdot p}{M_p M_\Lambda}\right)\right)$$

$$= |g|^2\frac{(E_p + M_p)}{M_p} - |g'|^2\,\frac{(M_p - E_p)}{M_p}$$

$$= \frac{9}{M_p}\left(|g|^2(E_p + M_p) + |g'|^2(E_p - M_p)\right) \times \frac{1}{(2\pi)^9}$$

We therefore have the result, using (10)-(14),

$$\iint d^3k\, d^3p\, \delta^{(4)}(Q-p-k)\ (\text{---})$$

$$= \int d^3p\ \delta(Q_0-p_0-k_0)\quad (\text{--})$$

$$= \int 4\pi p^2 dp\ \delta(M_\Lambda - E_p - \omega_\pi)\ (\text{--})$$

$$= \frac{4\pi p^2}{|J|}(\text{-}) = \left(4\pi\ p^2\ \frac{E_p\omega_\pi}{2pM_\Lambda}\right)(\text{--}) \quad \text{---}(15)$$

Substituting (15) in (9) we get

$$(2\pi)^3 \times \left(\tfrac{1}{2}\right) \times (2\pi)^4 \times\ \frac{1}{2\omega_\pi}\left(\frac{M_p}{E_p}\right)|m_{fi}|^2$$

$$\times\ 4\pi\, p^2\ \frac{E_p\omega_\pi}{2pM_\Lambda}$$

$$= (2\pi)^8\ p\left(\frac{M_p}{2M_\Lambda}\right)|m_{fi}|^2$$

$\therefore$ The partial decay rate

$$= (2\pi)^8\left(\frac{pM_p}{M_\Lambda}\right)\frac{1}{2}\,\frac{1}{(2\pi)^9}\left(|g|^2+|g'|^2\left(\frac{E_p-M_p}{E_p+M_p}\right)\right)\left(\frac{E_p+M_p}{M_p}\right)$$

$$= \left(\frac{1}{4\pi}\right)\left(\frac{|\vec{p}|}{M_\Lambda}\right)\left(|g|^2(E_p+M_p)+|g'|^2(E_p-M_p)\right)$$