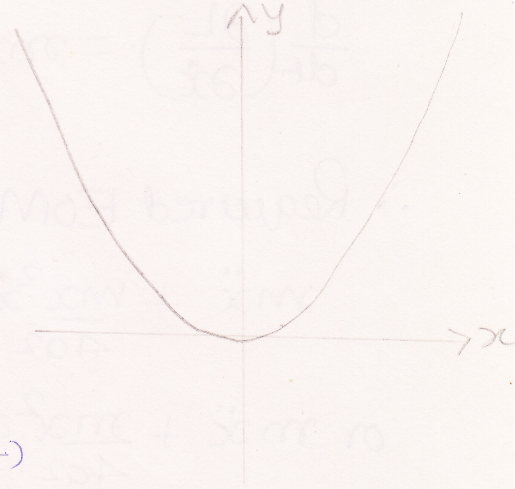
Lagrangian Formalism — Motion on Curves.

A particle slides on a smooth wire in shape of a parabola in vertical plane. Set up the Lagrangian and obtain the equation of motion.

Let the oy axis be chosen vertically upwards. The equation of the parabola can be written in the form



$$x^2 = 4ay \qquad \text{--- (1)}$$

The kinetic energy is given by

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \qquad \text{--- (2)}$$

and the potential energy by

$$V = mgy \qquad \text{--- (3)}$$

At this stage either x or y must be eliminated. Since specifying x, specifies the position uniquely, we choose x as the coordinate to be used and eliminate y.

We get from (1).

$$2x\dot{x} = 4a\dot{y} \Rightarrow \dot{y} = x\dot{x}/2a \qquad \text{--- (4)}$$

$$\therefore T = \frac{1}{2}m\left[\dot{x}^2 + \frac{(x\dot{x})^2}{4a^2}\right] \qquad V = mgx^2/4a$$

Hence we get

$$L = \frac{1}{2}m\left[\dot{x}^2 + \left(\frac{x\dot{x}}{2a}\right)^2\right] - \frac{mg}{4a}x^2$$

or $L = \frac{1}{2} m\dot{x}^2 + \frac{1}{8a^2} m x^2 \dot{x}^2 - \frac{mgx^2}{4a}$

EOM ~~are~~ is obtained from

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} + \frac{m}{4a^2} x^2 \dot{x} \qquad \frac{\partial L}{\partial x} = \frac{mx}{4a^2}\dot{x}^2 - \frac{mgx}{2a}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} + \frac{m}{4a^2}\left(x^2\ddot{x} + 2x\dot{x}^2\right)$$

∴ Required EOM is

$$m\ddot{x} + \frac{mx^2\ddot{x}}{4a^2} + \frac{m}{4a^2} 2x\dot{x}^2 - \frac{mx}{4a^2}\dot{x}^2 + \frac{mgx}{2a} = 0$$

or $$m\ddot{x} + \frac{mx^2\ddot{x}}{4a^2} + \frac{m}{4a^2} x\dot{x}^2 + \frac{mgx}{2a} = 0.$$