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Complex Variables

Principles and Problem Sessions

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KOKUYO

No.

Date

To the author.

How do you do? I am a retired math teacher of high school in Japan. I have ever taught elementary complex number that corresponds your texbook chapter one, sometimes introduced $e^{i\theta} = \cos\theta + i\sin\theta$ using Taylor expansion.

When I studied complex analysis in my college forty years more ago, I was interested in the Residue theorem but couldn't understand well.

I'm now retired, have full of time to study math, I found your book last December in Tokyo.

Your commentary is far different from other books.

Manytime you explain one by one very kindly, but sometime you dash at full speed.

No.

Date

I love your description and appreciate your original style.

I found some misprints and wrong answer especially in the problem sessions.

I want your text book would be popular among many students all over the world. When you revise the book for improvement, I want you would examine them with your staff. My indications may be wrong, if you find out, please point out my mistakes.

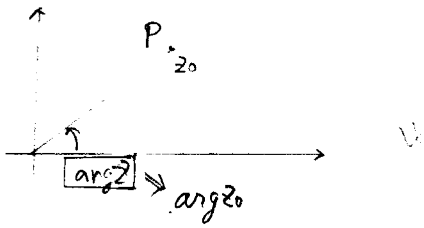
I have only finished chap 1, 2, 4. 5 and 6. Hopefully I want to read through. I am frustrated at my lack of ability and feel sad when there are difficult problems that I can't solve. Hope you would publish some guide book or student solution manual.

Best Regards. Takashi Matsuoka March 26. 2014.

Chap 1.

§1.1.4. Representation of angles (p5).

Fig. 1.2.



☑ 1.2.4. Problem (p9).

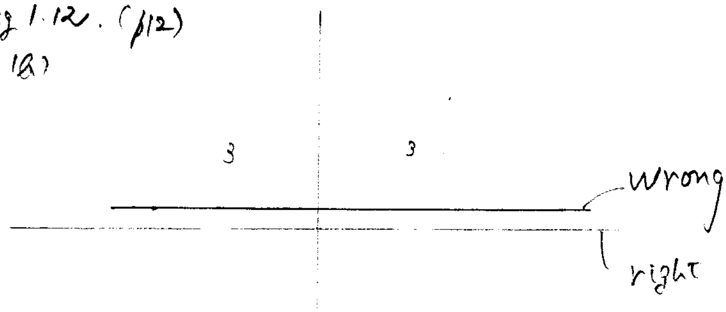
(p9) $\arg(z_1-z_2) - \arg\boxed{(z_4-z_3)} = \pm\frac{\pi}{2}$

⇘ $z_3 - z_4$

☑ 1.2.7. Short examples.

Fig 1.12. (p12)

(a)



The centers of five circles must be on the perpendicular bisector of the line joining two points z_1 and z_2.

§§1.1. Exercises (p274)

the first table

z $0<\theta<2\pi$. $\boxed{\frac{\pi}{2}}<\theta<\frac{3}{2}\pi$ $\Downarrow$ $-\frac{\pi}{2}$

$(1-i)^5$

§§1.2 Exercises (p275)

[1]

(c) $\arg(z-1)=\frac{\pi}{2}$, $|z+2|=\boxed{3}$ $\Downarrow$ 5

Answers (p277)

[1]

(b) $\boxed{1+7i}$ $\Downarrow$ $1+4i$

§§1.4. Tutorial

[1] (f). p280

$\arg(z_0-3)=\boxed{i-\alpha}$ $\Downarrow$ $-\alpha$, $|z_0-3|=d$

[2] (p281)

angles $\boxed{BAO}$ $\Downarrow$ AOB, OAB, OBA, BMO, $\boxed{CAB}$ $\Downarrow$ AOM and $\boxed{DBO}$ $\Downarrow$ CAB

§§1.7. Exercises

(2) Answer (p286)

$\boxed{\bar{\lambda} z_1 + \lambda \bar{z}_2 + 2c = 0}$, $\lambda = \frac{a+ib}{2}$

$\Downarrow$

$\bar{\lambda} z^* + \lambda z + c = 0$

§§1.8 Mixed

p289.

$\boxed{[3]}$ In sample answer 2, ...

$\searrow$ delete

$\boxed{[4]}$

$\searrow$ [3]

§§1.9 Mixed Bag

[1] (p290) $\boxed{1.10 \ (p32)}$

$\downarrow$

1.3.1 (p16)

[2] $\boxed{|z| > \mathrm{Re}\, z}$, $\boxed{|z| > \mathrm{Im}\, z}$

$\Downarrow$ $\Downarrow$

$|z| \geq \mathrm{Re}\, z$ $|z| \geq \mathrm{Im}\, z$

[4] (p290)

$$\left|\sum_{k=1}^{n} \xi_k \eta_k\right|^2 = \left|\sum_{k=1}^{n} |\xi_k|^2\right| \left|\sum_{k=1}^{n} |\eta_k|^2\right| - \boxed{\left|\sum_{1\le k<j\le n} |\xi_k^* \eta_k - \xi_k \eta_k^*|^2\right|}$$

delete

Wrong

$\Downarrow$

$$\sum_{k=1}^{n} |\xi_k|^2 \sum_{k=1}^{n} |\eta_k|^2 - \sum_{1\le k<j\le n} |\bar{\xi}_k \eta_j - \eta_k \bar{\xi}_j|^2$$

No. 4

Date

[11]. (p291)

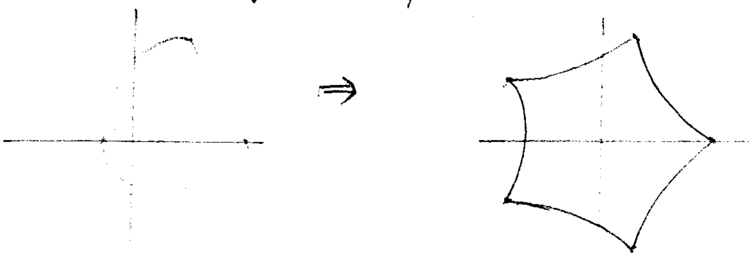
$x = a\cdot\coth\alpha$, $\quad R = \boxed{a|\operatorname{cosech}\alpha|}$

$\Downarrow$

$|a|\cdot|\operatorname{cosech}\alpha|$

[15]. (p292)

(b) Fig 1.13. Hypocycloid for $n=5$



[16] (p292).

in the family of circles $\boxed{1.8}$ p $\boxed{28}$ ...

$\boxed{1.8} \Rightarrow 1.2.8$, $\boxed{28} \Rightarrow 13$

... in $\boxed{1.7\,(d)}$ p $\boxed{28}$

$\boxed{1.7\,(d)} \Rightarrow 1.2.7$, $\boxed{28} \Rightarrow 12$

Answers

(15) (p293)

~~(a) The equation for the epicycloid is~~

$$\cancel{z = \frac{R}{n}\{(n+1)\exp(i\theta) - \exp[i(n+1)\theta]\}}$$

$$\Downarrow$$

$$\cancel{z = \frac{R}{n}\{(n+1)\exp(i\theta) - \exp[-i(n+1)\theta]\}}$$

[16] (p293)

discussed in Ch $\boxed{3}$.

$$\Downarrow$$

9

Chap 2.

§2.2

p30

$$\exp(iN\phi) = \exp(i\alpha) \quad (2.26)$$

implying that $N\phi = \alpha + \underbrace{2\pi i m}_{\Downarrow\, 2\pi m}$, $m \in \mathbb{Z}$

[v] 2.3.3 Problem

Solution (p33)

(1) $2? = \frac{2\pi}{3} i + \underbrace{2n\pi}_{\Downarrow\, 2n\pi i} \longrightarrow z = (n + \frac{1}{3})\pi i$

§2.5

Continuity of a function (p36)

(i) $\lim_{\boxed{z \to 0}} f(z)$ does not exist, or

$\Downarrow$

$z \to z_0$

(ii) $\lim_{\boxed{z \to 0}} f(z)$

$\Downarrow$

$z \to z_0$

§2.6.1. Cauchy Rieman equations in polar form (p38)

$$\frac{1}{r}\frac{\partial u(r,\theta)}{\partial \theta} = -\underbrace{\frac{\partial v(r,\theta)}{\boxed{\partial \theta}}}_{\Downarrow\, \partial r} \quad (r \neq 0) \quad (2.32)$$

2.7.3. Problem

Solution (p41)

(a)

(2.47) $|f(z)-f(0)| = r\,|(\cos^3\theta-\sin^3\theta)+i(\cos^3\theta+\sin^3\theta)|$

$= r\sqrt{2(\cos^6\theta+\sin^6\theta)}$

$\le \boxed{2r^3}$

$\Downarrow$

$2r$

the following description must be changed..

(b) (p41)

$$\lim_{z\to 0}\frac{f(z)-f(0)}{z-0} = \wedge\ \frac{1}{x+iy}\left(\frac{x^3-y^3}{x^2+y^2}+i\,\frac{x^3+y^3}{x^2+y^2}\right) \qquad (2.48)$$

missed $\Downarrow$

$$\lim_{z\to 0}\frac{1}{x+iy}\left(\frac{x^3-y^3}{x^2+y^2}+i\,\frac{x^3+y^3}{x^2+y^2}\right).$$

§2.9 Harmonic functions

Theorem 2.2. (p45)

analytic in a domain $\boxed{\text{if}}$ and only if

$\Downarrow$

if

2.9.1. Problem (p46)

Integrating Eq (2.65) w.r.t. y, we get

$\Downarrow$

(2.62)

§2.10. Graphical Representation of Functions

p47 (l.5) imaginary parts of $z^2=(x^2-y^2)\underbrace{-}_{\Downarrow\ +}2ixy$

$$x^2-y^2=c_1,\quad \underbrace{xy}_{\Downarrow\ 2xy}=c_2$$

p47 (l.19) we take $f(z)=\dfrac{z-z_1}{\underbrace{\boxed{z+z_2}}_{\Downarrow\ z-z_2}}$

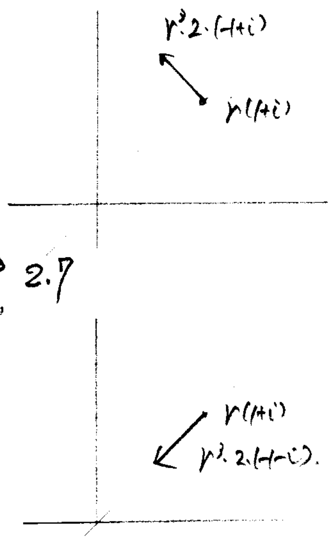
p48.

About the Polya representation

Let $z=r(1+i)$, $z^3=r^3 2(-1+i)$

and $\bar{z}^3=r^3 2(-1-i)$

So your description 「Figure $\boxed{2.6} \Rightarrow 2.7$ shows the Polya field representation of z^3, and the Polya field of $\bar{z}^3$ is shown in Fig. $\underbrace{\boxed{2.7}}_{\Downarrow\ 2.6}$」 is wrong.



§§2.1 Exercises. (p295)

[3] (b) $\cosh(z_1+z_2)\cdot\cosh(z_1-z_2) = \underline{\cosh^2 z_1 + \cosh^2 z_2}$

$\Downarrow$

$\cosh^2 z_1 + \cosh^2 z_2 - 1$

§§2.2 Exercises

Answer (p297)

[1](k) $\underline{\operatorname{cosech} z = \dfrac{\sin x\cdot\cosh y - i\cos x\cdot\sin y}{\sinh^2 x + \sin^2 y}}$

$\Downarrow$

$\operatorname{cosec} z = \dfrac{\sin x\cdot\cosh y - i\cos x\cdot\sinh y}{\sin^2 x + \sinh^2 y}$

[2] (b) $\underline{z = \pm n\pi}$

$\Downarrow$

all values of z.

§§2.4. Exercise (p297)

[2] (b) Answer $\underline{(2n\pi \pm 3)i}$

$\Downarrow$

$2n\pi \pm 3i$

(c) Answer $n\pi \underline{\pm} (-1)^n \ln(\sqrt{5}-2)\, i$

$\Downarrow$

$n\pi - (-1)^n i \ln(\sqrt{5}-2)$

or $n\pi + (-1)^n i \ln(\sqrt{5}+2)$

(e) Answer $\frac{1}{2}\ln\left(\frac{4}{3}\right) + (n+\frac{1}{2})\pi i$ (p300)

$$\Downarrow$$

$$\frac{1}{2}\ln\left(\frac{4}{3}\right) + n\pi i$$

(f) Answer $(n+\frac{1}{4})\pi \pm \frac{i}{2}\ln 2$ (p300)

$$\Downarrow$$

$$(2n+\frac{1}{2})\pi \pm i\ln 2$$

p300 (14) [3] numbering missed

$$\Downarrow$$

[4]

[4] (a) Answer $2n\pi \pm i\alpha$

$$\Downarrow$$

$$(2n\pi \pm \alpha)i$$

§§ 2.6. Tutorial

[3] Answer (p302)

(c) $\exp\left(\frac{\pi i}{6}\right) \times \{1, r_5, r_5^2, r_5^3, r_5^4\}$ where $r_5 = \exp\left(\frac{2\pi}{5}i\right)$

$$\Downarrow$$

$$\sqrt[5]{2}\exp\left(\frac{\pi i}{6}\right).$$

[4] If you leave the order of the question from (a) to (i), you must change the order of the column in the answer.

questions (a, b, c, d, e, f, g, h, i)

corresponds to the answer

(a, d, g, b, e, h, c, f, i)

§§2.8 Exercise

[2]

(b) Answer (p306)

The straight line $y=mx$ is $\frac{1-m^2}{1+im^2}$, which ...

$\Downarrow$

$\frac{1-m^2}{(1+im)^2}$

~~(c) Answer (p306)~~

~~as well as along all the straight line $y=mx$ are all equal.~~

§§2.10 Quiz

(b) $f(x,y)=\frac{x^3-iy^3}{x^2+y^2}$ (p308)

$\Downarrow$

$$f(x,y)=\begin{cases}\frac{x^3-iy^3}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$

(d) Answer

CR equations obeyed $\quad \underline{\text{Yes}}$

$\Downarrow$

No

§§ 2.11 (p309)

(c). Answer.

$\underline{\text{Not analytic anywhere}}$

$\Downarrow$

Analytic everywhere

(d) Answer

$\underline{\text{Analytic everywhere}}$

$\Downarrow$

Not analytic anywhere

(k) Answer (p310)

$z = \underline{n\pi}$

$\Downarrow$

n

§§ 2.12 (p311).

[3] (d) Answer.

$f(z) = i\cos z, \quad v(x,y) = \underline{\cos x . \sinh y}$

$\Downarrow$

$\cos x . \cosh y$

§§ 2.13 Mixed Bag

(10) (a) Answer (p 314)

$$i^i = \underline{\exp\left(-2\pi n - \frac{\pi}{4}\right)}.$$

$$\Downarrow$$

$$\exp\left(-2\pi n - \frac{\pi}{2}\right)$$

(18). p314,

$$\frac{\partial u(r,\theta)}{\partial r} = \frac{1}{r}\cdot\frac{\partial v(r,\theta)}{\partial \theta}, \quad \frac{1}{r}\cdot\frac{\partial u(r,\theta)}{\partial \theta} = \underline{-\frac{\partial v(r,\theta)}{\partial \theta}} \quad (r \neq 0)$$

$$\Downarrow$$

$$-\frac{\partial v(r,\theta)}{\partial r}$$

Chap 4

§ 4.1.1 Short examples

(d) p 82

(l 3) $$\frac{e^{kx}}{\cosh^2 x} = \frac{\boxed{e^{kx}} \Rightarrow 4 e^{kx}}{\exp(2x)+\exp(-2x)+2} \le \boxed{e^{(k-2)x}} \Rightarrow 4e^{(k-2)x}$$

(l 6) $$\frac{e^{kx}}{\cosh^2 x} = \frac{\boxed{e^{kx}} \Rightarrow 4 e^{kx}}{\exp(2x)+\exp(-2x)+2} \le \boxed{e^{(k-2)x}} \Rightarrow 4e^{(k-2)x}$$

☑ 4.3.2. Problem (p 85)

. . . . (4.36)

with $f(z) = z^2 + \underbrace{\lambda \bar{z}^2}_{\Downarrow\ \lambda\bar{z}}$ for the present problem.

$$= \int_0^{b} [(-2ay + \lambda y) + i(a^2 - y^2 + \underbrace{\lambda y}_{\Downarrow\ \lambda a})]\, dy \qquad (4.39)$$

(p 86) $$-\int_{AD} f(z)\,dz = -\int_0^{b} \underbrace{(-y^2 + i\lambda y)}_{\Downarrow\ -y^2 - i\lambda y}(i\,dy) \qquad (4.43)$$

§4.4.1 Jordan's lemma

(p 88) Let $C_R^{\uparrow}$ $\underbrace{(C\bar{R})}_{\Downarrow\ C_R^{-}}$ be the semicircle $|z| = R$ . . .

(p 89) $$\le 2M \frac{\pi}{2kR} \underbrace{\left(1 - \exp(-2kR)\right)}_{\Downarrow\ 1-\exp(-kR)} \qquad (4.58)$$

§4.5.3 Short examples

p92, l7.

$$\left|\int_{C_R^+}\right| \le M\cdot R^{m-n}\max(e^{-ky})\times\pi R \le \underbrace{MR^{m+1-n}}_{\Downarrow \; \pi MR^{m+1-n}}$$

§4.5.4. Short examples (p92)

(b) Let $\underbrace{\gamma}_{\Downarrow\;\gamma_2}$ be the arc of the circle $|z|=R$

§4.6.1. Cauchy's theorem (p92)

Theorem 4.3

be a function such that $\underbrace{\frac{\partial f}{\partial z}}_{\Downarrow\;\frac{df}{dz}}$ exists and

§4.6.3. Indefinite integral

p96 the bottom of the page

$\frac{df}{dz}=-iz^2 \longrightarrow f(z)=\underbrace{-\frac{i}{z}}_{\Downarrow\;\frac{i}{z}}+c$ (4.84)

§4.7.1. Short examples

(b) (p98)

where the extra second term $\int_{BCA}$, . . . result in

§$\underbrace{4.6\,(a)}_{\downarrow}$

§4.5.3(a) $\nearrow \int_{-R}^{R} e^{ikx}Q(x)\,dx$

$$\int_{-\infty}^{\infty} e^{ikx}Q(x)\,dx = \lim_{R\to\infty}\int_{-R}^{R} e^{ikx}Q(x)\,dx = \lim_{R\to\infty}\int_{AB} e^{ikz}Q(z)\,dz$$

$$= \lim_{R\to\infty}\int_{AB} \underbrace{e^{ikx}}_{\Downarrow\; e^{ikz}}Q(z)\,dz + \lim_{R\to\infty}\int_{BCA} e^{ikz}Q(z)\,dz$$

§ 4.7.2. Translation and rotation of the contour

(p99. ℓ5) $\displaystyle\int_{-\infty}^{\infty} \exp\Big(\underbrace{-(x-ik)^2}_{\Downarrow \; -(x-\frac{ik}{2})^2}\Big)dx = \int_{\mathrm{Im}\,t=-\frac{k}{2}} \exp(-t^2)\,dt$ (4.94)

(p99. ℓ8) $\displaystyle\int_{\mathrm{Im}\,t=\underbrace{\frac{k}{2}}_{\Downarrow \; -\frac{k}{2}}} \exp(-t^2)\,dt = \int_{\mathrm{Im}\,t=0} \exp(-t^2)\,dt$ (4.95)

§§4.1. Questions (p337)

(3) Answer

$b - c < 1$, $\underline{2b - c - 1 > 1}$.

$\Downarrow$

$2b - c - 1 > 0$

(8). Answer $\underline{a < b}$

$\Downarrow$

$b > a > 0$, $b < a < 0$

§§4.3 Exercise

[1] Answer (p340)

$\underline{\ln 2} - \frac{\pi}{4} i$

$\Downarrow$

$\frac{1}{2} \ln 2 - \frac{\pi}{4} i$

[2] Answer (p340)

(a) $\frac{-2}{3}$

$\Downarrow$

0

(b) $\underline{-a^3}$, $\frac{2}{3} a^3$

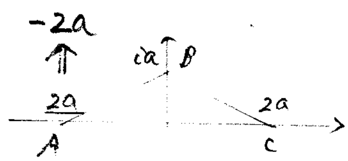
$\Downarrow$

$\frac{13}{3} a^3$

the bottom of the page (p339) Fig. $\underline{4.1}$

$\Downarrow$

4.3



[3] Answer (p340)

$\underline{(b)\ -(13+16i)/3}$

$\Downarrow$

$2 + 4i$

(5). Answer $\dfrac{16\pi i}{}$ $\Downarrow$ $16i$

(6). Answer $\dfrac{2\pi i R^n \cdot z_0^{n-1}}{}$ $\Downarrow$ $2\pi i R^2 \cdot n\, \bar{z}_0^{\,n-1}$

(7) Answer (a) $\dfrac{0}{}$ $\Downarrow$ $\dfrac{-2(-1)^n \cdot a^{2n+1}}{2n-1}$

(10). Answer $\dfrac{\sqrt{\pi}\,(4-i)/2}{}$ $\Downarrow$ $\dfrac{\sqrt{\pi}}{4\sqrt{2}}\,(1-3i)$

§§7.4 Questions

p341 the bottom of the page Fig $\underline{4.2}$ $\Downarrow$ 4.4

p342 column heading of table $\underline{\text{ANMK}}$ $\Downarrow$ ANMA

Answer

	ANMA	KCLK	AKNCDA	ABCMLA	KNCMLCK	PQRSP
$\frac{z^4}{\cosh 100z}$						$\underline{\text{No}}$ $\Downarrow$ Yes
$\frac{z^2}{z^2-\sqrt{2}z+1}$	$\underline{\text{Yes}}$ $\Downarrow$ No	$\underline{\text{Yes}}$ $\Downarrow$ No	$\underline{\text{No}}$ $\Downarrow$ Yes			
$\exp(1/z)$					$\underline{\text{No}}$ $\Downarrow$ Yes	

§§4.5 Exercise (p343)

(3)

$$\int_0^{2\pi} \frac{d\phi}{a^2\cos^2\phi + b^2\sin^2\phi} = \frac{\pi}{2ab} \Downarrow \frac{2\pi}{ab}$$

§§4.6 Exercise

p344 the bottom of the page

Fig $4.3 \Downarrow 4.5$, Fig $4.4 \Downarrow 4.6$

[2] Answers (p345)

(a) $I_{\Gamma_5} = (-16+i)a^2/2 - (8+2i)a$

$\Downarrow$

$(-16+i)a^2/2 - 2ia$

(b) $I_{\Gamma_5} = (\frac{1}{\pi} - 1)(e^{2a\pi} - 1) + e^{2a\pi}(1 - e^{-i\pi a})$

$\Downarrow$

$\frac{1}{\pi}(2 \cdot e^{2\pi a} - 1)(1 - e^{-\pi a i})$

(c). $I_{\Gamma_5} = I_{\Gamma_6} = \frac{1}{3}((ia-3)^3 - 9$

$\Downarrow$

$\frac{1}{3}((ia-3)^3 + 9$

(d) $I_{\Gamma_5} = I_{\Gamma_6} = i\sin z$

$\Downarrow$

$i\sin a$

(3) Answer

(1) $2+i(e-\frac{1}{e})/2$

$\Downarrow$

$2+i(e-\frac{1}{e})$.

(2) $1-i$

$\Downarrow$

$\frac{1}{\pi}(-1+i)$.

§§4.7. Tutorial

p346) the bottom of the page Fig 4.5

$\Downarrow$

7

p347) [12] For $a>b>0$. the number of the question

$\Downarrow$

[2]

$$\int_0^\infty \exp(-px^2)\sin ax \sin bx\,dx = \frac{1}{4}\sqrt{\frac{\pi}{p}}\left\{\exp\left[-\frac{(a-b)^2}{4p}\right]-\exp\left[\frac{(a+b)^2}{4p}\right]\right\}$$

$\Uparrow$ $\exp\left[-\frac{(a+b)^2}{4p}\right]$

$$\int_0^\infty \exp(-px^2)\cos ax \cos bx\,dx = \frac{1}{4}\sqrt{\frac{\pi}{p}}\left\{\exp\left[-\frac{(a-b)^2}{4p}\right]+\exp\left[\frac{(a+b)^2}{4p}\right]\right\}$$

$\Downarrow$

$\exp\left[-\frac{(a+b)^2}{4p}\right]$

§§4.8 Tutorial

(a)

$$\int_{ACB} \exp(ipz^2)dz = \frac{iR}{2}\int_0^{\frac{\pi}{4}} \exp[ipR^2\cos 2\theta - pR^2\sin 2\theta]\,d\theta$$

$\Downarrow$

$$iR\int_0^{\frac{\pi}{4}} \exp[ipR^2\cos 2\theta - pR^2\sin 2\theta]\cdot e^{i\theta}d\theta$$

(b)

$$\left|\int_{ACB} \exp(ipz^2)\,dz\right| \le \underline{\int_0^{\frac{\pi}{2}} \exp(-pR^2\theta)\,d\theta}$$

$$\Downarrow$$

$$\frac{R}{2}\int_0^{\frac{\pi}{2}} \exp(-pR^2\tfrac{2}{\pi}\theta)\,d\theta$$

the bottom of the page   Fig 4.$\underline{6}$

$\Downarrow$

4.8

§§4.9. Exercise (p350)

[1] $\underline{\int_0^\infty e^{-q^2x^2}\sin^2 ax} = \underline{\frac{1}{2}}\cdot\frac{\sqrt{\pi}}{q}\left[1-\exp\left(-\frac{a^2}{q^2}\right)\right]$

$\Downarrow$ $\Downarrow$

$\int_0^\infty e^{-q^2x^2}\sin^2 ax\,dx$ $\quad \frac{1}{4}$

[2] $\underline{\int_0^\infty e^{-q^2x^2}\begin{bmatrix}\sin p(x+c)\\ \cos p(x+c)\end{bmatrix}} = \underline{\frac{\sqrt{\pi}}{q}}\exp(-p^2/4q^2)\begin{pmatrix}\sin pc\\ \cos pc\end{pmatrix}$

$\Downarrow$ $\Downarrow$

$\int_0^\infty e^{-q^2x^2}\begin{bmatrix}\sin p(x+c)\\ \cos p(x+c)\end{bmatrix}dx$ $\quad \frac{1}{2}\cdot\frac{\sqrt{\pi}}{q}$

[3] $\int_0^\infty x\cdot e^{-q^2x^2}\cdot\sin ax\,dx = \sqrt{\pi}\,\underline{\left(\frac{a}{4q^2}\right)}\exp\left(-\frac{a^2}{4q^2}\right)$

$\Downarrow$

$\frac{a}{4q^3}$

[5] $\int_0^\infty x^3 e^{-q^2x^2}\sin ax\,dx = \sqrt{\pi}\cdot\underline{\frac{6aq^2-q^3}{16q^7}}\exp\left(-\frac{a^2}{4q^2}\right)$

$\Downarrow$

$\frac{6aq^2-a^3}{16q^7}$

It's very sad that I can't find any clue to solve the problem from (6) to (17).

§§4.10. Exercise
the bottom of the page (p352)
Fig 4.7
$\Downarrow$
4.9

§§4.11. Mixed Bag

p354 (2). Answer (a) $f(z) = \underline{-i \operatorname{cosec}^2 z}$
$\Downarrow$
$i \cdot \cot z$

(6)

I think this problem is correct, but I suppose
p353 the original may be $\left| \int_\Gamma \frac{z^2+z+1}{z^3+1} dz \right| < \frac{3}{4}\pi$

(9).
p354 where $\Gamma = \{ z \mid \underline{|z-1| < \rho}, \ \varepsilon < \arg(z-1) < 2\pi - \varepsilon \}$
$\Downarrow$
$|z-1| = \rho$

$\underline{\omega \to 0.}$
$\Downarrow$
$\rho \to 0$

(8) (p354)

Show that the integral $\displaystyle\int_C \frac{a^z}{z^2}\,dz$ vanishes.

$$\Downarrow$$

$$\int_C \frac{a^{-z}}{z^2}\,dz$$

Though I am not sure about my opinion, I think your insistence is wrong.

Chapter 5

§5.1

p113, ℓ10.

$$\oint_\Gamma \frac{f(z)}{z-z_0}dz = \oint_\gamma \frac{f(z)}{z-z_0}dz = \oint_\gamma \phi(z)dz + \oint_\gamma \frac{f(z_0)}{z-z_0}dz + f(z_0)\oint_{\underset{\Downarrow \\ \gamma}{\boxed{z}}} dz$$

$$= \oint_\gamma \phi(z)dz + f(z_0)\oint_\gamma \frac{dz}{z-z_0}\underset{\substack{\Downarrow \\ \text{no need}}}{dz}$$

¶5.1.1. Short examples p114

(a)

$$\frac{1}{\underset{\substack{\Downarrow \\ z^2}}{\boxed{z^3}}-5z+6} = \frac{1}{z-3} - \frac{1}{z-2}$$

$$\oint_\gamma \frac{e^z}{\underset{\substack{\Downarrow \\ z^2}}{\boxed{z^3}}-5z+6}dz =$$

(c) $$\oint_\gamma \frac{e^z}{\underset{\substack{\Downarrow \\ z^2}}{\boxed{z^3}}-5z+6}dz =$$

¶5.5.1. Short examples p119

(c).

$$\tan z = \underset{\substack{\Downarrow \\ a_1 z}}{\underline{a_1}} + a_3 z^3 + a_5 z^5 + a_7 z^7 + \cdots \qquad (5.32)$$

$$\sin z = (\underset{\substack{\Downarrow \\ a_1 z}}{\underline{a_1}} + a_3 z^3 + a_5 z^5 + a_7 z^7 \cdots)\cos z \qquad (5.33)$$

§5.6. Laurent expansion

Theorem 5.4. p121.

$$a_n = \frac{1}{2\pi i}\oint_C \frac{f(\zeta)}{(\zeta - z_0)^{n+1}} \underset{\Downarrow}{dz} \quad (5.42)$$

$$d\zeta$$

p122 l4.

$$f(z) = \frac{1}{2\pi i}\oint_{\Gamma_1} \frac{f(\zeta)}{\zeta - z} d\zeta + \frac{1}{\underset{\Downarrow}{2\pi}}\oint_{\Gamma_2} \frac{f(\zeta)}{\zeta - z} d\zeta \quad (5.45)$$

$$2\pi i$$

$$= -\frac{1}{2\pi i}\oint_{\Gamma_1} \frac{f(\zeta)}{\zeta - z} d\zeta + \frac{1}{\underset{\Downarrow}{2\pi}}\oint_{\Gamma_2} \frac{f(\zeta)}{\zeta - z} d\zeta \quad (5.46)$$

$$2\pi i$$

the last line of the page p122

$$a_n = \frac{1}{2\pi i}\oint_{\Gamma_2} \frac{f(\zeta)}{(\zeta - z_0)^{n+1}} \checkmark \text{ missed} \quad (5.50)$$

$$\Downarrow$$

$$\frac{1}{2\pi i}\oint_{\Gamma_2} \frac{f(\zeta)}{(\zeta - z_0)^{n+1}} d\zeta$$

□ 5.7.2 Problem p125

$$\frac{1}{z+2}=\frac{1}{2}\cdot\frac{1}{1+\frac{z}{2}}=\frac{1}{2}\left(1-\frac{z}{2}+\frac{z^2}{4}\underset{\substack{\Downarrow\\ -}}{+}\frac{z^3}{8}+\cdots\right) \qquad (5.59)$$

p126 l3.

$$\frac{1}{z-1}=\frac{1}{z}\frac{1}{1-\frac{1}{z}}=\frac{1}{z}\left(1\underset{\substack{\Downarrow\\ +}}{-}\frac{1}{z}+\frac{1}{z^2}\underset{\substack{\Downarrow\\ +}}{-}\frac{1}{z^3}+\cdots\right) \qquad (5.60)$$

No. 27

Date

No. 28

Date

§§5.1 (p355)

[4] a rectangle with the corners at $(2, \pm 3)$,

and $\underline{(\pm 2, 3)}$

$\Downarrow$

$(-2, \pm 3)$

Answer (9) $\underline{-\frac{1}{2}\pi i}$

$\Downarrow$

$\frac{1}{2}\pi i$

§§5.5 (p361).

(3) Use the answer in Q(2) to get $\underline{\text{four}}$ different

$\Downarrow$

three

convergence for the $\underline{\text{four}}$ cases.

$\Downarrow$

three

§§5.3 (p358)

(n) $\frac{1}{(1+z)^n} = 1 - nz + \underline{n(n+1)z^2} \underline{+} \cdots$

$\Downarrow$

$\frac{n(n+1)}{2} z^2 -$

§§ 5.6 p363

[1]

(e) $\dfrac{2z}{(z+1)(z+3)}$ $\Downarrow$ $\dfrac{2z+4}{(z+1)(z+3)}$

(f) $\dfrac{z}{(z+1)^2}$ $\Downarrow$ $\dfrac{1}{z(z+1)^2}$

[2] (b) $\dfrac{1}{z^2+5z+6}$, $z_0 = 1$ $\Downarrow$ -1.

Answer

[1]

Answer of Q(e) is (f) p364.

Answer of Q(f) is (e).

You have missed the order of the answer.

[2]

(a) $0 \le |z+1| < 1$; $-\Big[1 + \underbrace{(z+1)^{-2}}_{\Downarrow\,(z+1)^2} + \underbrace{(z+1)^{-3}}_{\Downarrow\,(z+1)^4} + \cdots\Big]$

(b) $0 \le |z+1| < 1$; $\dfrac{1}{2} - \dfrac{3(z+1)}{4} + \dfrac{7(z+1)^2}{8} \underbrace{+}_{\Downarrow\,-} \dfrac{15(z+1)^3}{16} + \cdots$ (minus sign)

$1 < |z+1| < 2$ $\dfrac{1}{z+1} - \dfrac{1}{(z+1)^2} + \dfrac{1}{(z+1)^3} - \cdots - \dfrac{1}{2} + \dfrac{z+1}{4} - \dfrac{(z+1)^2}{8} + \dfrac{(z+1)^3}{16} \underbrace{+}_{\Downarrow\,-} \cdots$ (minus sign) p364 (c). $|z+1|<\sqrt{3}$; $-\frac{1}{z+1}+\frac{1}{3}-\frac{5(z+1)}{9}+\frac{7(z+1)^2}{27}-\frac{(z+1)^3}{81}-\frac{(z+1)^4}{243}+\cdots$

$\Downarrow$

$+$

$|z+1|>\sqrt{3}$; $\frac{3}{(z+1)^2}+\frac{3}{(z+1)^3}-\frac{3}{(z+1)^4}-\frac{15}{(z+1)^5}+\cdots$

$\Downarrow$

$-$

p365 (d) $0 \lneq |z+\frac{2}{3}|<\frac{2}{3}$; $-\frac{3}{8}(z+\frac{2}{3})^{-1}-\frac{9}{32}-\frac{81}{128}(z+\frac{2}{3})+\frac{15\cdot 27}{16\cdot 32}(z+\frac{2}{3})^2+\cdots$

$\Downarrow$

$-$

in the case $\frac{2}{3}<|z+\frac{2}{3}|<\frac{4}{3}$:

$\left[-\frac{1}{4}(z+\frac{2}{3})^{-1}+\frac{1}{6}(z+\frac{2}{3})^{-2}+\frac{1}{9}(z+\frac{2}{3})^{-3}+\cdots\right]+\left[-\frac{1}{6}+\frac{2}{9}(z+\frac{2}{3})^2-\frac{8}{27}(z+\frac{2}{3})^3+\frac{32}{81}(z+\frac{2}{3})^4+\cdots\right]$

$\Downarrow$ $-\frac{1}{8}$

$\Downarrow$

$\frac{3}{32}-\frac{9}{128}(z+\frac{2}{3})+\frac{27}{512}(z+\frac{2}{3})^2-\frac{81}{2048}(z+\frac{2}{3})^3+\cdots$

in the case $|z+\frac{2}{3}|>\frac{4}{3}$. $-\frac{1}{4}(z+\frac{2}{3})^{-1}+\frac{1}{3}(z+\frac{2}{3})^{-3}-\frac{2}{9}(z+\frac{2}{3})^{-4}+\cdots$

$\Downarrow$

$\frac{1}{3}(z+\frac{2}{3})^{-3}-\frac{2}{9}(z+\frac{2}{3})^{-4}+\frac{4}{9}(z+\frac{2}{3})^{-5}\cdots$

(3) in case of $2<|z|<4$.

$-\left(\frac{1}{8}+\frac{1}{z^2}+\frac{2}{z^3}+\frac{4}{z^4}+\cdots\right)-\left(\frac{1}{8}+\frac{z}{32}+\frac{z^2}{128}+\frac{z^3}{512}+\cdots\right)$

$\Downarrow$

$\frac{1}{z}$

in case of $|z|>4$. $-\frac{1}{z^2}-\frac{6}{z^3}-\frac{28}{z^4}-\frac{120}{z^5}-\cdots$

$\Downarrow$

$\frac{1}{z^2}+\frac{6}{z^3}+\frac{28}{z^4}+\frac{120}{z^5}+\cdots$

§§5.7. p366.

Qu (7) $\dfrac{1}{\exp(\pi z)+1}$, $z_0 = \pi i$

$\Downarrow$

$\dfrac{1}{\exp z+1}$

Answer

(8) $\{z \mid \underbrace{n}_{\Downarrow\, n-1} < |z-2| < \underbrace{n-1}_{\Downarrow\, n}\}$; $n = 1, 2, 3 \dots$

(9) $\{z \mid |z-\pi i| < \underbrace{2}_{\Downarrow\, 2\pi}\}$, $\{z \mid \underbrace{2}_{\Downarrow\, 2\pi} < |z-\pi i| < \underbrace{4}_{\Downarrow\, 4\pi}\}$, $\dots$

(10). $\{z \mid \underbrace{\frac{n}{n+1}}_{\Downarrow\, \frac{n-1}{n}} < |z-1| < \underbrace{\frac{n-1}{n}}_{\Downarrow\, \frac{n}{n+1}}\}$; $n = 1, 2, 3 \dots$; $\{z \mid \underbrace{|z-2| > 2}_{\Downarrow\, |z-1|>1}\}$.

No 32

Date

§§5.8 p367.

Qu [9] $\frac{z}{\sin^3 z}$, $\underline{z_0 = n\pi}$ $\Downarrow$ 0

(13).

$$\underline{\exp\left[t\left(z+\frac{1}{z}\right)\right]} = \sum_{n=-\infty}^{\infty} z^n J_n(t).$$

$$\Downarrow$$

$$\exp\left[\frac{t}{2}\left(z-\frac{1}{z}\right)\right]$$

$$\underline{J_n(x) = \sum_{n=0}^{\infty} \frac{(-1)^k}{(k+n)!\,k!}\left(\frac{x}{2}\right)^{2n}}$$

$$\Downarrow$$

$$J_n(t) = \left(\frac{t}{2}\right)^n \sum_{\substack{k\geq 0\\ k+n\geq 0}} \frac{(-1)^k}{(k+n)!\,k!}\left(\frac{t}{2}\right)^{2k}$$

Answers (p368)

(6) $\operatorname{cosech} z = \frac{1}{z} \underset{\Downarrow\, +}{-} \frac{z}{6} + \frac{7z^3}{360} \underset{\Downarrow\, +}{-} \frac{31z^5}{15120} + \cdots$

(8). $\frac{z+3}{\sin^2 z} = \frac{3}{z^2} + \frac{1}{z} \underset{\Downarrow\, -}{+} \frac{z^2}{2} \underset{\Downarrow\, -}{+} \frac{z^3}{6} + \frac{z^6}{120} + \frac{7z^7}{360} \underset{\Downarrow\, -}{+} \cdots$

No. 3.3

Date

(9) $\dfrac{z}{\sin^3 z} = \dfrac{1}{z^2} + \dfrac{1}{2} + \dfrac{17z^2}{120} + \underset{\sim\sim}{\dfrac{457z^4}{15210}} + \cdots$

$$\Downarrow$$

$$\frac{457z^4}{15120}$$

(11) $\dfrac{1}{\exp z - 1} = \dfrac{1}{z} - \dfrac{1}{2!} + \dfrac{z}{12} - \underset{\sim\sim}{\dfrac{z^3}{702}} + \cdots$

$$\Downarrow$$

$$\frac{z^3}{720}$$

§§ 5.10. p390

(2).

$f(z) = \dfrac{1}{3z} \underline{\sum \left(\dfrac{z}{3}\right)^m} = \dfrac{1}{3z}\left(1 - \underline{\dfrac{1}{3}} + \dfrac{z^2}{3^2} \cdots\right)$

$\Downarrow \qquad \Downarrow$

$\sum \left(-\dfrac{z}{3}\right)^m \qquad \dfrac{1}{3z}\left(1 - \dfrac{z}{3} + \dfrac{z^2}{3^2} - \cdots\right)$

Chap 6

§6.4 p149

(l4) $\lim_{z\to 5} \dfrac{z-5}{\cosh z - \cosh 5} = \dfrac{1}{\sinh z}\Big|_{\underline{z=3n}} = \underline{-\dfrac{1}{\sinh 5}}$

$\Downarrow \qquad\qquad \Downarrow$

$\dfrac{1}{\sinh z}\Big|_{z=5} = \dfrac{1}{\sinh 5}$

§6.7 p154

☑ 6.7.1 Problem

(a) The circle $|3z-2|=1$, has the center at $z=\frac{2}{3}$ and radius $\underline{1}$.

$\Downarrow$

$\frac{1}{3}$

6.7.2. Short examples p155

(b).

(l4) $f=\frac{1}{w}\left(1\underline{-}\frac{aw}{2}\underline{+}\frac{1}{2}a^2w^2+\cdots\right)\left(1-\frac{bw}{2}\underline{+}\frac{1}{2}b^2w^2\underline{+}\cdots\right)$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

minus sign $\quad - \quad - \quad - \quad -$

(l5) $=\frac{1}{w}-\frac{a+b}{2}\,\underset{\sim}{+}\,\frac{1}{2}(a+b)^2w+\cdots$

$\Downarrow$

minus sign $-$

and the residue of $f(z)$ at infinity is given by $\underset{\sim}{-}\frac{1}{2}(a+b)^2$

$\Downarrow$

$+$

☑ 6.4.2 p149

(b) last line) $\frac{1}{z}\left[1+\left(\frac{z^2}{3!}\cdot\frac{z^4}{5!}+\cdots\right)+\left(\frac{z^2}{3!}\underset{\sim}{+}\frac{z^4}{5!}+\cdots\right)^2+\cdots\right]$

$\Downarrow$

$-$

p150 (l2) $\frac{1}{\pi z^2}\left[1+\left(\frac{\pi^2z^2}{3!}-\frac{\pi^4z^4}{5!}+\cdots\right)+\left(\frac{\pi^2z^2}{3!}\underset{\sim}{+}\frac{\pi^4z^4}{5!}+\cdots\right)^2+\cdots\right]$

$\Downarrow$

$-$

☑ 6.7.3 Problem p155

$$\mathrm{Res}\left\{\frac{dz}{3z^2+7z+3}\right\}_{z=z_+} = \frac{1}{\sqrt{13}}$$

$$\Downarrow$$

$$\mathrm{Res}\left\{\frac{1}{3z^2+7z+3}\right\}_{z=z_+}$$

☑ 6.8.2 Problem p158

(l 7) $f(z) = r^{(1-p)} \cdot \rho^p \underbrace{e^{i[(1-p\theta]}} e^{ip\phi} Q(z)$ (6.49)

$$\Downarrow$$

$$e^{i(1-p)\theta}$$

(last line) Eqs. $(6.\underline{52}) - (6.\underline{54})$ by $-2\pi i$

$$\Downarrow \qquad \Downarrow$$

$$53 \qquad 55$$

No. 36

Date

§§6.1. p323.

(19) $\underline{\sin\left(\frac{1}{z}+1\right)}$

$\Downarrow$

$\sin\left(\frac{1}{z+1}\right)$

§§6.2 p325.

(16) $\underline{\dfrac{z^4+1}{\exp(\pi z)+1}}$

$\Downarrow$

$\dfrac{z^4-1}{\exp(\pi z)+1}$

Answers p326

		location	order
(16).	Poles	$\underline{\text{all } m \text{ except } \pm 2}$,	1.

$\Downarrow$

$\pm\sqrt{N}\ (N\neq 4)$. $\pm\sqrt{N}\,i\ (N\neq 0)$

§§6.3 p328

IV $\underline{(26)}$ $\Downarrow$ (25).

V $\underline{(16)}$ $\Downarrow$ III. $\underline{(25)}$ $\Downarrow$ (26)

§§6.4. p380

Answers

(16). $\{ \underline{in}, \frac{1}{\pi} \}$

$\downarrow$

$i(n+\frac{1}{2})$.

(17) $\{ \underline{2n\pi}, 2 \}$

$\Downarrow$

$2n\pi i$

(18). $\{0, 1\}$; $\{ \underline{e^{\frac{2k\pi i}{5}}}, k=0, \cdots 4, -\frac{1}{5} \}$

$\Downarrow$

$e^{\frac{2k+1}{5}\pi i}$

(19) $\{ (2n+1)\pi i \}$ $\underline{-\operatorname{sech}(2n+1)\pi}$ $\}$.

$\Downarrow$

$-\cosh(2n+1)\pi$

§§6.5 p381.

(2). (d). $\underline{\dfrac{\cos(\pi z)^2}{(3z-1)^3}}$, at $z=\frac{1}{3}$

$\Downarrow$

$\dfrac{\cos^2(\pi z)}{(3z-1)^3}$

(7) p381

(b) $\dfrac{\sin(z)^3}{z^6}$, at $z=0$

$\Downarrow$

$\dfrac{\sin^3 z}{z^6}$

(c) $\dfrac{z^2}{\sin(z)^5}$, at $z=0$

$\Downarrow$

$\dfrac{z^2}{\sin^5 z}$

§§6.6. p383

Answers.

[2] (d) $\dfrac{\pi^2}{10}\operatorname{cosec}\left(\dfrac{\pi}{20}\right)$

$\Downarrow$

$-\dfrac{\pi^2}{5} i \operatorname{cosec}\left(\dfrac{\pi}{20}\right)$

(3) (g) $\dfrac{\pi i}{3}$

$\Downarrow$

$\dfrac{\pi i}{60}$

(h) $\sqrt{8}\,\pi i$

$\Downarrow$

$-\sqrt{8}\,\pi i$

(4) $\pi\alpha$
$\Downarrow$
$-\pi\alpha$.

§§6.7. p384

Answers (7). E
$\Downarrow$
B

§§6.8 p385.

(1) (c)

$$J = \oint \frac{dz}{6z^2+13z+6}$$
$\Downarrow$
$$-i\oint \frac{dz}{6z^2+13z+6}$$

(d)

$$\int_0^{2\pi} \frac{d\theta}{13+12\cos\theta} = \frac{2\pi}{\sqrt{7}}$$
$\Downarrow$
$$\frac{2\pi}{5}$$

No. 40

Date

§§6.9 p386.

(3). $\int_0^{\pi} \frac{d\theta}{3-\sin\theta}$

$\Downarrow$

$\int_0^{2\pi} \frac{d\theta}{3-\sin\theta}$

(8). $\int_0^{2\pi} \frac{d\theta}{(a+b\cos^2\theta)^2} = \frac{\pi(2a+b)}{a^{\frac{3}{2}}(a+b)^{\frac{5}{2}}}$, $a>0.\ b>0.\ \left|\frac{a}{b}\right|>1$

$\Downarrow$

$\frac{\pi(2a+b)}{a^{\frac{3}{2}}(a+b)^{\frac{3}{2}}}$

(9). $\int_0^{2\pi} \sin^{2m}\theta' d\theta$

$\Downarrow$

$\int_0^{2\pi} \sin^{2m}\theta\, d\theta$

(11) $\int_0^{2\pi} \frac{\cos^2 2\theta}{1-2p\cos\theta+p^2} d\theta = \pi \frac{1+p^4}{1-p^4}$, $0<p<1$

$\Downarrow$

$\pi \frac{1+p^4}{1-p^2}$

(14) $\displaystyle\int_0^{\pi} \frac{\cos n\theta \cdot \cos\theta}{1-2a\cos\theta+a^2}\, d\theta = \begin{cases} \dfrac{\pi}{2} \cdot \dfrac{1+a^2}{1-a^2}\, a^{n-1}, & \underline{a<1} \Rightarrow |a|<1 \\ \dfrac{\pi}{2}\, \dfrac{a^2+1}{a^2-1}\, \dfrac{1}{a^{n+1}}, & \underline{a>1} \Rightarrow |a|>1. \end{cases}$

Answers

(1) $\underline{\frac{3}{4}\pi}$

$\Downarrow$

$\frac{2}{3}\pi$

§§ 6.11. p328

Answers

(1) $\underline{-\frac{5}{2}}$

$\Downarrow$

0

(2) $\underline{0}$

$\Downarrow$

$-\frac{5}{2}$

(3) $\underline{2i \sinh(\pi\alpha)}$

$\Downarrow$

$-\frac{i}{e} \sinh(\pi\alpha)$

No 42

Date

§§6.12 p389

(1).

$$f(z) = \frac{1}{z^2(z-3)} = \frac{1}{z^3(1-\frac{3}{z})}$$

$$= \frac{1}{z^3} \times \left(1 - \frac{3}{z} + \frac{9}{z^2} + \cdots \right)$$

$\Downarrow$

$+$

(2)

$$g(z) = \frac{3}{2z} - \frac{9}{4z^2} + \cdots + (-1)^n \frac{3^n}{2^n z^n} + \cdots$$

$\Downarrow$

$$(-1)^{n-1} \frac{3^n}{2^n z^n}$$

(the lost second line) $\cosh(\frac{1}{z})$

$\Downarrow$

$\sinh(\frac{1}{z})$

3.8.6.13. p391.

(3) (d) $\dfrac{z^{10}+tz^{5}+1}{z^N(z-a)(\underline{z+b})}$, at $z=0$ for $N>1$.

$\Downarrow$

$\dfrac{z^{10}+tz^{5}+1}{z^N(z-a)(z-b)}$

Answers p393.

(8) (a) $\dfrac{\dfrac{b_0 a_1 - b_1 a_0}{b_0^2}}{}$

$\Downarrow$

$\dfrac{b_1 a_0 - b_0 a_1}{b_0^2}$

(b) $\dfrac{\dfrac{a_0}{b_0}}{}$

$\Downarrow$

$-\dfrac{a_0}{b_0}$