Indian Institute of Technology Bhubaneswar
School of Basic Sciences

Spring Mid-Semester Examination-2017

Subject Name : Statistical Mechanics	**Subject Code : PH5L008**

Date : 21 February 2017

Duration : 2 Hours **Full Marks : 30**

Answer a new question on a fresh page (strictly) and all parts of a question together

(1) The canonical partition function of some kind of particles is given by,

$$Q(T,V,N) = \left(\frac{V-Nb}{\Lambda^3}\right)^N \exp\left(\frac{aN^2}{Vk_BT}\right),$$

where,

$$\Lambda = \frac{h}{\sqrt{2\pi m k_B T}};$$

a, and b are constants; other symbols have their usual meaning.

(i) Find the internal energy $U(T,V,N)$. **(2 Marks)**

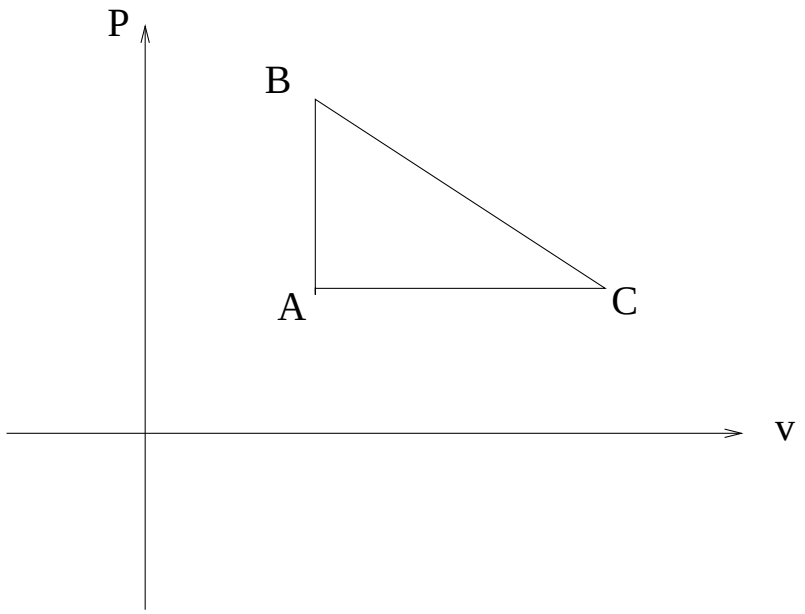
(ii) Find the entropy, $S(T,V,N)$. **(2 Marks)**

(iii) Does the expression for S, provide a valid fundamental relation[1]? If not, what is wrong with S ? How can Q be corrected? **(2 Marks)**

(Total : 6 Marks)

[1]except perhaps at $T = 0$.

(2) A system of $\boldsymbol{n = 100}$ moles of an ideal gas is taken through a **quasi-static reversible** cyclic process : $\boldsymbol{A \to B \to C \to A}$ depicted below on the Pressure-Volume phase plane.



Let

$\boldsymbol{V(A) = 3}$ M$^{\mathbf{3}}$ $\quad \boldsymbol{P(A) = 10^5}$ pa

$\boldsymbol{V(C) = 6}$ M$^{\mathbf{3}}$ $\quad \boldsymbol{P(B) = 2 \times 10^5}$ pa

Let $\boldsymbol{D}$ be the point on the phase diagram at which the temperature is maximum. Find the pressure, volume, and temperature at $\boldsymbol{D}$. The entropy at $\boldsymbol{A}$ is taken as zero : $\boldsymbol{S(A) = 0}$. Find the value of entropy at $\boldsymbol{D}$.

$\boldsymbol{C_V = 3nR/2}$; $\boldsymbol{C_P = 5nR/2}$; $\boldsymbol{R = 8.31447}$ joules (kelvin)$^{-\mathbf{1}}$ (mol)$^{-\mathbf{1}}$ **(12 Marks)**

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(3) Consider a closed system of $\boldsymbol{N}$ non-interacting point particles at temperature $\boldsymbol{T}$ kelvin. Let $\boldsymbol{\epsilon = k_B T}$. These particles occupy three non-degenerate energy levels :

ground state of energy zero;

first excited state of energy $\boldsymbol{\epsilon}$ joules and

second excited state of energy $\boldsymbol{2\epsilon}$ joules.

The (canonical ensemble) average of energy is $\boldsymbol{10^{25}\epsilon}$ joules. The particles are identical and **distinguishable**[2]. What is the value of $\boldsymbol{N}$? **(6 Marks)**

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(4) Let $\boldsymbol{\{E_n = n\epsilon : n = 0, 1, 2, \cdots\}}$, be the energy levels of a macroscopic closed system in equilibrium at $\boldsymbol{T = 300}$ kelvin, where $\boldsymbol{\epsilon = 300\ k_B}$ joules. The $\boldsymbol{n}$-th energy level is $\boldsymbol{(n+1)}$-fold degenerate. Calculate the entropy of the system. Write your answer to third decimal accuracy. ($\boldsymbol{k_B = 1.381 \times 10^{-23}}$) **(6 Marks)**

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[2]The $\boldsymbol{N}$ particle partition function is given by the single-particle partition function raised to the power $\boldsymbol{N}$: $\boldsymbol{Q_N = Q_1^N}$**.** Since the particles are distinguishable we do not divide by $\boldsymbol{N!}$.