Phy 523
PARTICLE PHYSICS
Solutions for problem sheet VIII

36. Consider a massless Dirac particle. Show that $(1+\gamma_5)v(p)$ represents an antiparticle of negative helicity. $v(p)$ is a plane wave solution $\not{p}v(p) = 0$

Solution:

We obtain the wave functions of a particle(antiparticle) by considering the wave functions of a massive Dirac particle and taking the limit mass $m \to 0$. The wave function of a particle of momentum $\vec{p}$, energy E and spin $\vec{s}$ is given by

$$u(p, \vec{s}) = (E+m)^{1/2} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}.\vec{p}}{E+m}\chi_s \end{pmatrix}$$

and for the antiparticle we have

$$v(p, \vec{s}) = (E+m)^{1/2} \begin{pmatrix} \frac{\vec{\sigma}.\vec{p}}{E+m}\chi_{-s} \\ \chi_{-s} \end{pmatrix}$$

Here $\vec{\sigma}.\vec{s}\chi_s = \chi_s$. For helicity eigenstates we choose $\vec{s}$ along the direction of $\vec{p}$ for positive helicity states and opposite to $\vec{s}$ for negative helicity states.The wave function for negative helicity massless antiparticle we have (using $\vec{\sigma}.\vec{p}\chi_{-s} = |p|\vec{\sigma}.(\overrightarrow{-s})\chi_{-s} = |p|\chi_{-s}$ and $|p| = E$),

$$v(\vec{p},\ \text{negative helicity}) = \sqrt{E} \begin{pmatrix} \chi_{-s} \\ \chi_{-s} \end{pmatrix}$$

where we have used the massless condition $|p| = E$ In our representation

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Thus

$$(1+\gamma_5)v(\vec{p}, \text{negative helicity}) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \chi_{-s} \\ \chi_{-s} \end{pmatrix}$$

$$= 2 \begin{pmatrix} \chi_{-s} \\ \chi_{-s} \end{pmatrix}$$

Thus $(1+\gamma_5)/2$ projects negative helicity antiparticle.

37. What are the helicities of $\bar{u}(p)(1+\gamma_5), \bar{u}(p)(1-\gamma_5), \bar{v}(1+\gamma_5)$ and $\bar{v}(1-\gamma_5)$, where the spinors are for massless particles.

Solution:
the negative helicity particle is given by

$$u(p,\ \text{negative helicity}) = \frac{(1-\gamma_5)}{2} u(p, \vec{s})$$

We have

$$\bar{u}(p, \text{negative helicity}) = u(p, \text{negative helicity})^\dagger \gamma^0 = u(p, \vec{s})^\dagger (1-\gamma_5)^\dagger \gamma^0$$

$$= u(p, \text{negative helicity})^\dagger \gamma^0 (1+\gamma_5) = \bar{u}(p, \text{negative helicity})(1+\gamma_5)$$

Thus $\bar{u}(p)(1+\gamma_5)$ represents negative helicity. Similarly $\bar{u}(p)(1-\gamma_5)$ represents positive helicity. $\bar{v}(p)(1+\gamma_5)$ and $\bar{v}(p)(1-\gamma_5)$ represent final state wave function for an antiparticle with positive and negative helicity respectively.

38. The field $\psi(x)$ of a massive Dirac particle which is its own antiparticle is referred to as Majorana particle. It obeys the condition $\psi^c(x) = \psi(x)$ where $\psi^c(x) = C\bar{\psi}(x)^T$. C is the charge conjugation operator $C = \gamma^0\gamma^2$. Show that $\bar{\psi}(x)\gamma^\mu\psi(x) = \bar{\psi}(x)\sigma^{\mu\nu}\psi(x) = 0$

We have already seen earlier (Problem 14) that

$$\bar{\psi}^c(x)\gamma^\mu\psi^c(x) = -\bar{\psi}(x)\gamma^\mu\psi(x)$$

Using the Majorana condition $\psi^c(x) = \psi(x), \bar{\psi}^c(x) = \bar{\psi}(x)$ we get for the left hand side, $\bar{\psi}(x)\gamma^\mu\psi(x)$ which is equal to its negative value (right hand side). This is possible only if each side is equal to zero.

Identical arguments hold for $\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)$.

39. Show that if electron neutrino is a Majorana particle neutrinoless double beta decay of nucleus is a possibility.

Solution:

We have in a nuclear β^- decay, $X(Z, A) \to Y(Z+1, A) + e^- + \bar{\nu}_e$. We can also have $\nu_e + Y(Z+1, A) \to W(Z+2) + e^-$. Since for a Majorana particle we have the anitneutrino is the same as neutrino the emitted electron antineutrino in the beta decay of X can be abosrbed by Y to form W. Thus we have $X(Z, A) \rightarrow W(Z+2, A)+e^{-}+e^{-}$, which is neutrinolesss beta decay. Notice the neutrino if massless would be right handed and so the absorption of Y to form W will not occur. However if the neutrino is massive then the concept of chirality is not valid and the aborption can take place. Thus the double beta decay matrix element would be proportional to the mass of the neutrino.

40. If we neglect the mass of the electron, what will be final helicity of the elctron if it's initial helicity is negative in (a) Rutherford scattering (b) Compton scattering

Solution:

In the massless limit the interaction term $\bar{\psi}(x)\gamma^{\mu}\psi(x)$ preserves helicity. To see this consider an initial left handed wave function $(1-\gamma_5)\psi(x)$. Then the interaction term is $\bar{\psi}(x)\gamma^{\mu}(1-\gamma_5)\psi(x) = \bar{\psi}(x)(1+\gamma_5)\gamma^{\mu}\psi(x)$. We have seen the final wave function $\bar{\psi}(x)(1+\gamma_5)$ describes a negative helicity state. Thus in both the cases (Rutherford scattering and Compton scattering) the final state would have negative helicity.