QFT-15 Solved Problem

Processes Allowed by an Interaction Term

Q(3) QFT-2016 End Semester Examination

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The original four fermion interaction for beta decay of neutron

$$n \longrightarrow p + e^- + \bar{\nu}$$

is of the of form

$$\bar{\psi}_p(x)\gamma_\mu\psi_n(x)\bar{\psi}_\nu(x)\gamma^\mu\psi_e(x) + h.c.$$

Now consider other processes given below. Which of these processes (real or virtual) are permitted and which ones are not permitted by the above interaction in the first order?

- $\bar{p} \longrightarrow \bar{n} + e^- + \bar{\nu}$;
- $\bar{p} \longrightarrow \bar{n} + e^- + \nu$;
- $n \longrightarrow p + e^+ + \nu$;
- $p \longrightarrow n + e^+ + \bar{\nu}$;
- $\bar{n} \longrightarrow \bar{p} + e^+ + \nu$;
- $\bar{n} \longrightarrow \bar{p} + e^+ + \bar{\nu}$.

Give brief reason in each case.

DRAFT : To be checked for typographical errors

☺*Solution*: The action of different field operators is as follows.

- ψ_n: Destroys a neutron or creates an anti-neutron.
- ψ_p: Destroys a proton or creates an anti-proton.
- ψ_e: Destroys an electron or creates an positron.
- ψ_ν:Destroys a neutrino or creates an anti-neutrino.
- $\bar{\psi}_n$ Destroys an anti-neutron or creates an neutron
- $\bar{\psi}_p$ Destroys an anti-proton or creates a proton

- $\bar{\psi}_e$ Destroys an positron or creates an electron
- $\bar{\psi}_\nu$ Destroys an anti-neutrino or creates a neutrino

Writing the hermitian conjugate of the term explicitly written we get a sum of two terms.

$$\bar{\psi}_p(x)\gamma_\mu\psi_n(x)\bar{\psi}_\nu(x)\gamma^\mu\psi_e(x) + \bar{\psi}_n(x)\gamma_\mu\psi_p(x)\bar{\psi}_e(x)\gamma^\mu\psi_\nu(x).$$

	Process	Fields required	Allowed or Not?
(a)	$\bar{p} \longrightarrow \bar{n} + e^- + \bar{\nu}$	$\psi_p, \bar{\psi}_n, \bar{\psi}_e, \psi_\nu$	Allowed
(b)	$\bar{p} \longrightarrow \bar{n} + e^- + \nu$	$\psi_p, \bar{\psi}_n, \bar{\psi}_e, \bar{\psi}_\nu$	Not Allowed
(c)	$n \longrightarrow p + e^+ + \nu$	$\psi_n, \bar{\psi}_p, \psi_e, \bar{\psi}_\nu$	Not Allowed
(d)	$p \longrightarrow n + e^+ + \bar{\nu}$	$\psi_p, \bar{\psi}_n, \psi_e, \psi_\nu$	Not Allowed
(e)	$\bar{n} \longrightarrow \bar{p} + e^+ + \nu$	$\bar{\psi}_n, \psi_p, \bar{\psi}_e, \psi_\nu$	Allowed
(f)	$\bar{n} \longrightarrow \bar{p} + e^+ + \bar{\nu}$	$\psi_p, \bar{\psi}_n, \psi_e, \psi_\nu$	Not Allowed

We are looking for answer in lowest order only. So we can complete the above table by looking at the creation and annihilation operators in the field operators.

In fact that the Hamiltonian is hermtian means you can take particle from initial state and put it as anti particle in the final state and vice versa. So under this transformation an allowed process will remain allowed; a process, obtained from a process that is not allowed, will also not be permitted.

Assuming that the form of the Hamiltonian for beta decay has been written correctly, you can also use charge conservation and lepton number conservation to cross check the answers.