Quantum Field Theory 2016- Final Examination

Q1(a) For a real free Klein Gordon field, mass m, compute

$$\langle 0|\phi(x)\phi(y)|\vec{k},\vec{q}\rangle$$

and show that the result is properly symmetrized wave function for two identical bosons with momenta $\vec{q},\vec{p}$. Here $|\vec{k},\vec{q}\rangle$ is the state with two bosons with momenta $\vec{k}.\vec{q}$

We need to evaluate the matrix element of $\phi(x)\phi(y)$ between the vacuum and two particle state $|\vec{k},\vec{q}\rangle$ which can be expressed as

$$|\vec{k},\vec{q}\rangle = N \cdot a^{+}(k)\, a^{+}(q)|0\rangle$$

Where N is a normalization constant which need not be fixed for present purpose.

Note that we can write the two creation operators in any order because their commutator is zero. Therefore we consider

$$\Psi(x,y) = \langle 0|\, \phi(x)\, \phi(y)\, a^{+}(k)\, a^{+}(q)|0\rangle$$

move

$$= \langle 0|\, \phi(x) \left\{ a^{+}(k)\phi(y) + [\phi(y), a^{+}(k)] \right\} a^{+}(q)|0\rangle \qquad \text{---(1)}$$

Recall that $\phi(y) = \int \frac{d^3p}{2\omega_p}\left(f_p(y)\,a(p) + f_p^{*}(y)\,a^{+}(p)\right)$ (2)

and $[a^{+}(p), a^{+}(k)] = 2\omega_p\, \delta(\vec{p}-\vec{k})$. (3)

$f_k(x)$ denotes free particle wave function for a spin zero scalar field [Notation: Gasiorowicz]

$$\therefore \quad [\phi(y), a^+(k)] = f_k(y) \qquad (4)$$

Using (4) in one (1) gives

$$\Psi(x,y) = \langle 0| \phi(x)\{a^+(k)\,\phi(y) + f_k(y)\}a^+(q)|0\rangle$$

$$= \langle 0| \phi(x)\, a^+(k)\,\phi(y)\,a^+(q) + \phi(x)\, f_k(y)\, a^+(q)|0\rangle$$

Next we move $a^+(q)$ and $a^+(k)$ as indicated

These operators then act on $\langle 0|$ giving zero and we are left with commutator part alone.

$$\therefore \Psi(x,y) = \langle 0| f_k(x)\,\phi(y)\,a^+(q) + f_q(x) f_k(y)|0\rangle$$

$$= \langle 0| f_k(x)\, f_q(y) + f_q(x) f_k(y)|0\rangle$$

$+\langle 0|a^+(q)\cdots$ and $\langle 0|a^+(k)$ terms which become zero.

$$\therefore \Psi(x,y) = \left(f_k(x) f_q(y) + f_q(x) f_k(y)\right)$$

which shows that the wave function is correctly symmetrized wave function of two identical bosons.

Normalization constant in $|\vec{k},\vec{q}\rangle = N\, a^+(k)\, a^+(q)|0\rangle$ can be fixed by demanding

$$\langle \vec{k},\vec{q}\,|\,\vec{k}'\,\vec{q}\,'\rangle = \frac{1}{2}\left(\delta(\vec{k}-\vec{k}')\,\delta(\vec{q}-\vec{q}\,') + \delta(\vec{k}-\vec{q}\,')\,\delta(\vec{q}-\vec{k}')\right)$$

$$N = \frac{1}{\sqrt{2}}.$$