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Solved Example

(Ge)neralized potential

(Poin)t charge in electric and magnetic field

For a point charge moving with velocity $\vec{v}$ in (pre)sence of electric and magnetic field $\vec{E}$, $\vec{B}$ the (Lo)rentz force on the charged particle is

$$\vec{F} = q\left\{-\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} + \frac{1}{c}\vec{v}\times(\vec{\nabla}\times\vec{A})\right\}$$

(Wh)ere ϕ and $\vec{A}$ are scaler and vector potential (wh)ich give $\vec{E}$ & $\vec{B}$ by

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

(W)e want to determine generalized potential for (th)is system.

(Re)call!

$$\sum_\alpha \vec{F}^{(a)}_\alpha \frac{\partial \vec{r}_\alpha}{\partial q_j} = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\frac{\partial U}{\partial \dot{q}_j}$$

(I)n our present case

$$q \leftrightarrow \vec{r}_\alpha = (x, y, z)$$

α not present (only one part...)

If U is generalized potential we must (have)

$$F_x = -\frac{\partial U}{\partial x} + \frac{d}{dt}\frac{\partial U}{\partial \dot{x}} \; ; \quad |||^{ly} \text{ for } F_y \text{ and } F_z.$$

(w)here $U \equiv U(\vec{r}, \dot{\vec{r}}, t)$

$$F_x = +qE_x + \frac{q}{c}(\vec{v}\times\vec{B})_x$$

$v_y B_z$

$$= -q\frac{\partial\phi}{\partial x} - \frac{q}{c}\frac{\partial A_x}{\partial t} + \frac{q}{c} v_y \frac{\partial A_y}{\partial x} - v_y\frac{\partial A_x}{\partial y}$$

$$+ \frac{q}{c} v_z \left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right)$$

B_x

$$+ \frac{q}{c} v_x \frac{\partial A_x}{\partial x} - \frac{q}{c} v_x \frac{\partial A_x}{\partial x}$$

Add & Substract

$$= -q\frac{\partial\phi}{\partial x} + \frac{q}{c}\frac{\partial}{\partial x}(v_x A_x + v_y A_y + v_z A_z)$$

$$- \frac{q}{c}\frac{\partial A_x}{\partial x}v_x - \frac{q}{c}\frac{\partial A_x}{\partial y}v_y - \frac{q}{c}\frac{\partial A_x}{\partial z}v_z$$

(rearrange)

Note $v_x = \dot{x}$ etc

$$= -q\frac{\partial\phi}{\partial x} + \frac{q}{c}\frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \frac{q}{c}\frac{d}{dt}A_x$$

$$= \text{----} + -\frac{q}{c}\frac{d}{dt}\frac{\partial}{\partial v_x}(A_x v_x) + \text{--}$$

$$= \text{-----} + -\frac{q}{c}\frac{d}{dt}\frac{\partial}{\partial v_x}(\vec{A}\cdot\vec{v}) + \text{---}$$

$$= -q\frac{\partial\phi}{\partial x} + \frac{q}{c}\frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \frac{q}{c}\frac{d}{dt}\frac{\partial}{\partial\dot{x}}(\vec{A}$$

ϕ does not depend on $\dot{x}$

$$\therefore \frac{\partial\phi}{\partial\dot{x}} = \frac{\partial}{\partial\dot{x}}\phi = 0$$

$$F_x = -q\frac{\partial}{\partial x}\left(\phi - \frac{1}{c}\dot{\vec{r}}\cdot\vec{A}\right) + \frac{q}{c}\frac{d}{dt}\left(\frac{\partial}{\partial\dot{x}}\left(\phi - \frac{1}{c}\dot{\vec{r}}\right.\right.$$

If we define $U = \phi - \frac{1}{c} \dot{r} \cdot A$

$$F_x = -q \frac{\partial U}{\partial x} + \frac{d}{dt} \frac{\partial}{\partial \dot{x}} (\vec{\dot{r}} \cdot \vec{A})$$

and the Lagrangian takes the form

$$L = T - U$$
$$= \tfrac{1}{2} m \vec{\dot{r}}^2 + q \phi - \frac{q}{c} \vec{A} \cdot \vec{\dot{r}}$$

Exercise: Use the Lagrangian to derive the equation of motion of the charge particle in $\vec{E}$ & $\vec{B}$ fields

Question: What will be the Lagrangian for many charged particles moving in external electric and magnetic fields