University of Hyderabad
School of Physics

M.Sc.-I/IMSc.-III | <u>Quantum Mechanics</u> | **May 31, 2018**

May 14-Jul 6 (2018) | **MM: 10**

Quiz-I- Solutions

(a) Obtain energy eigen functions for a particle in one dimension restricted to move on half line, represented by potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ 0, & \text{if } x \geq 0. \end{cases} \tag{1}$$

(b) Are the energy eigenvalues degenerate? If yes, what is the degeneracy of energy eigenvalues?

(c) For the above system, write the most general solution of the time dependent Schrodinger equation.

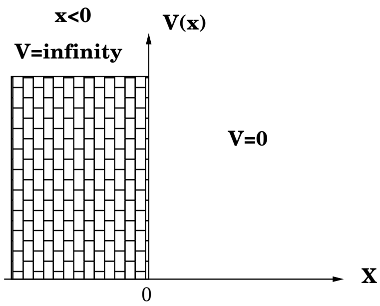

Fig. **1**

☺*Solution*: First of all there are no solutions for energy less than the absolute minimum of the potential, *i.e.* for $E < 0$. So we need to consider $E > 0$ only.

(a) Since the potential is infinite for $x < 0$, the wave function must vanish for $x < 0$.

For $x \geq 0$, the Schrodinger equation takes the form

$$-\frac{\hbar^2}{2m}\frac{d^2u(x)}{dx^2} = Eu(x) \tag{2}$$

$$\frac{d^2u(x)}{dx^2} + k^2u(x) = 0, \qquad k^2 = \frac{2mE}{\hbar^2}. \tag{3}$$

A general solution of the above equation can be written as

$$u(x) = A\sin kx + B\cos kx. \tag{4}$$

The wave function must be continuous at $x = 0$. This means that

$$u(x)|_{x=0} = 0 \Longrightarrow B = 0. \tag{5}$$

There is no other requirement at $x = 0$[1]. Hence the solution is $u_k(x) = A\sin kx, \quad k > 0$.

(b) There is only one linearly independent solution and the energy values are non-degenerate.

(c) A solution of the time dependent Schrodinger equation with energy is

$$\psi_k(x,t) = A\sin(kx)\,\exp\big(-iE_kt/\hbar\big), \quad k > 0.$$

Forming superposition with energy, or k, dependent coefficients gives the most general solution of the time dependent Schrodinger equation as

$$\Psi(x,t) = \int_0^\infty A(k)\,\sin(kx)\,\exp\big(-iE_kt/\hbar\big)\,dk, \quad \text{where } k > 0 \text{ and } E_k = \frac{\hbar^2k^2}{2m}. \tag{6}$$

Remark: The coefficients $A(k)$ in (6) can be determined if the wave function at time $t = 0$ is known.

[1] In particular, recall that at the point of infinite jump,(at $x = 0$), in the potential there is no restriction on the derivative of the solution.