22.6.2004

EoM; Newtonian Mechanics

CM005 (1)

Plot the Morse potential

$$V(x) = V_0\left(e^{-2x/\alpha} - e^{-x/\alpha}\right)$$

and find the period of small oscillations for a particle having energy E and moving a force field described by the Morse potential.

<u>Plot of Morse potential</u>

$$V(x) = V_0\left(e^{-2x/\alpha} - 2e^{-x/\alpha}\right)$$

as $x \to \infty$ $V(x) \to 0$.

as $x \to -\infty$ $e^{-2x/\alpha} > 2e^{-x/\alpha}$ and $e^{-2x/\alpha} \to \infty$.

$\therefore V(x) \to \infty$.

$$V(x) = 0 \Rightarrow e^{-2x/\alpha} - 2e^{-x/\alpha} = 0.$$

$$e^{-x/\alpha} - 2 = 0$$

$$\therefore -x/\alpha = \log 2 \Rightarrow -x = \alpha \log 2$$

or $x = -\alpha \log 2$.

At $x = -\alpha\log 2$ and $x \to \infty$ $V(x) = 0$.

$\therefore$ There is a maximum or a minimum in between

$$\frac{dV}{dx} = 0 \quad -\frac{2}{\alpha}e^{-2x/\alpha} + \frac{2}{\alpha}e^{-x/\alpha} = 0$$

$$\Rightarrow e^{-2x/\alpha} - e^{-x/\alpha} = 0$$

$$\Rightarrow e^{-x/\alpha} = 1 \Rightarrow x = 0$$

$$\frac{d^2V}{dx^2} = V_0\left(\frac{4}{\alpha^2}e^{-2x/\alpha} - \frac{2}{\alpha^2}e^{-x/\alpha}\right) \Rightarrow \left.\frac{d^2V}{dx^2}\right|_{x=0} = \frac{2V_0}{\alpha^2}.$$

$\therefore$ The potential has a minimum at $x = 0$.

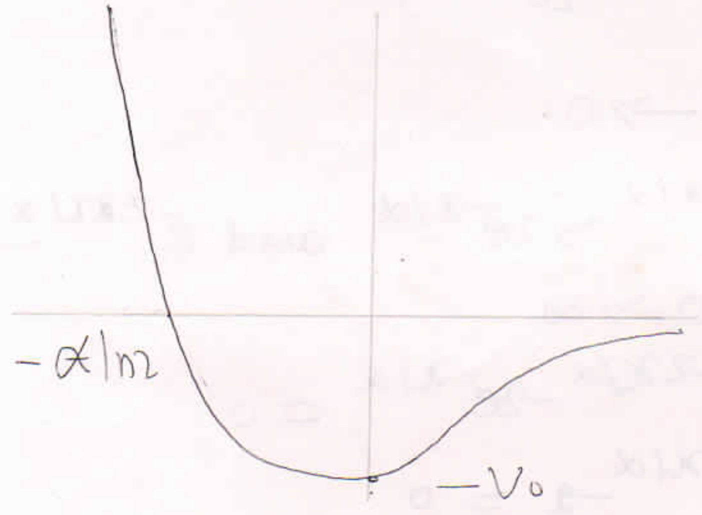
At $x=0$ $V(x) = -V_0$.

Also $V(x) = V_0 (e^{-2x/\alpha} - 2e^{-x/\alpha})$

$= V_0 e^{-x/\alpha} (e^{-x/\alpha} - 2)$

$V(x) > 0$ if $e^{-x/\alpha} - 2 > 0$ or $-x/\alpha > \log 2$ ie $x < \alpha \log 2$

$V(x) < 0$ if $e^{-x/\alpha} - 2 < 0$ or $x > \alpha \log 2$.



Also $V(x)$ ~~decreases~~ monotonically as x increases ~~decreases~~ for $x < 0$.

The equation of motion is

$$m\ddot{x} = -\frac{dV}{dx} = -V_0\left(-\frac{1}{\alpha}\right)\left(e^{-2x/\alpha}.2 - 2e^{-x/\alpha}\right)$$

$$= \frac{2V_0}{\alpha}\left(e^{-2x/\alpha} - e^{-x/\alpha}\right)$$

For small oscillations

$$m\ddot{x} \simeq -\frac{2V_0}{\alpha}\cdot\left(\frac{x}{\alpha}\right)$$

compare with $\ddot{x} + \omega^2 x = 0$

$$\omega^2 = \frac{2V_0}{\alpha^2 m} \implies \omega = \sqrt{\frac{2V_0}{m}}\,\frac{1}{\alpha}$$

$$\therefore T = \frac{2\pi}{\omega} = \sqrt{\frac{2m}{V_0}}\left(\frac{\pi}{\alpha}\right).$$