Q10
§§7.4

To evaluate $\int_0^\infty \frac{x \sin ax}{(x^2+b^2)^2+c^2} dx$ by the method of contour integration

Follow the notation and method of the previous question Q9/§§7.4.

The given integral in this case is

$$\frac{1}{2} \mathrm{Im} \oint_{ADBCA} \frac{z e^{iaz}}{(z^2+b^2)^2+c^2} dz$$

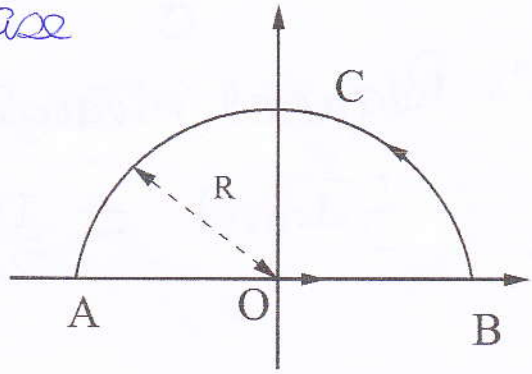

Fig. 1 Semi-circular contour

The poles are at

$\xi_1 = B+iA$, $\xi_2 = -B+iA$

$\xi_3 = -iA-B$ $\xi_4 = -iA+B$

Only ξ_1 and ξ_2 are enclosed by the contour in the upper half plane.

Residue at $\xi_1 = \xi_1 e^{ia\xi_1} \lim_{z \to \xi_1} \frac{(z-\xi_1)}{(z^2+b^2)^2+c^2}$

$$= \frac{\xi_1 e^{ia\xi_1}}{4(\xi_1^2+b^2)\xi_1} = \frac{e^{ia\xi}}{4ic}$$

Similarly the residue at ξ_2 is

$$= \frac{e^{ia\xi_2}}{(-4ic)}$$

$$\therefore J = \frac{(2\pi i)}{4ic}\left(e^{ia\xi_1} - e^{-ia\xi_2}\right)$$

$$= \left(\frac{\pi}{2c}\right)\left(e^{-aA+iaB} - e^{-aA-iaB}\right)$$

$$= \frac{\pi i}{c} e^{-aA} \sin aB$$

$\therefore$ Required integral

$$= \frac{1}{2} I_m J = \frac{\pi}{2c} e^{-aA} \sin aB.$$

$\boxed{20.5.2017}$