$\boxed{\text{§§7.4 Q2}}$ Compute the integral $\int_0^\infty \frac{\cos px\,dx}{(x^2+1)^2}$, $p>0$ by the method of contour integration.

Following the same procedure as in Q(1), we see that

$$\int_0^\infty \frac{\cos px\,dx}{(x^2+1)^2} = \frac{1}{2}\mathrm{Re}\lim_{R\to\infty}\int_{-R}^{R}\frac{e^{ipz}}{(z^2+1)^2}\,dz$$

$$= \frac{1}{2}\mathrm{Re}\lim_{R\to\infty}\oint_{AOBCA}\frac{e^{-ipz}}{(z^2+1)^2}\,dz$$

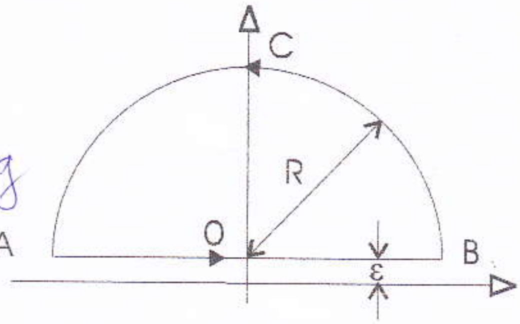

Fig. 1 Semi-circular-contour

The last integral is computed by making use of Cauchy Residue Theorem. Only the double pole at $z=i$ is enclosed inside the contour.

Therefore

$$\oint_{AOBCA}\frac{e^{ipz}}{(z^2+1)^2}\,dz = (2\pi i)\times\frac{d}{dz}(z-i)^2\frac{e^{ipz}}{(z^2+1)^2}\Big|_{z=i}$$

$$= (2\pi i)\times\frac{d}{dz}\frac{e^{ipz}}{(z+i)^2}\Big|_{z=i}$$

$$= (2\pi i)\times\frac{ip\,e^{ipz}(z+i)^2 - 2(z+i)e^{ipz}}{(z+i)^4}\Big|_{z=i}$$

$$= (2\pi i)\times\frac{ip\,e^{-pz}(2i)^2 - 4i\,e^{-p}}{(2i)^4} = \left(\frac{\pi i}{8}\right)e^{-p}(-4ip-4i)$$

$$= \frac{\pi}{2}(p+1)e^{-p}$$

$$\therefore \int_0^\infty \cos px/(x^2+1)^2\,dx = \frac{\pi}{4}(p+1)e^{-p}.$$ $\boxed{11\text{-}5\text{-}2017}$

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