VS-03 Problem Set

Linear Functionals

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@iitbbs.ac.in; akkhcu@gmail.com

[1] Let ψ be a linear functional on vector space $\mathbb{R}^3$. It is given that

$$\psi(x_1) = 3, \psi(x_2) = 4, \psi(x_3) = 5$$

where x_1, x_2, x_3 are the three vectors

$$x_1 = (1, 1, 0); \qquad x_2 = (0, 1, 1); \qquad x_3 = (1, 1, 1);$$

Find $\psi(y)$ where $y = (8, 9, 7)$

[2] A number of functionals are defined on vectors spaces as given below. In each case check if the functional is a linear functional.

(a) Functional Φ_1 on $\mathbb{R}^3$ is defined by $\Phi_1(\vec{A}) = \vec{A} \cdot \vec{A}$.

(b) Functional Φ_2 on $\mathbb{R}^3$ is defined by $\Phi_2(\vec{A}) = \vec{m} \cdot \vec{A} + \alpha$, where $\vec{m}$ is a fixed vector and α is real number. Is there any value of α for which $\vec{\Phi}_2$ is a linear functional ?.

(c) Φ_3 on $\mathbb{R}^n$ is defined by $\Phi_3 f = \sum_{k=1}^{n} |\xi_k|$, where $f = (\xi_1, \xi_2, ..., \xi_n)$.

(d) For $p(t)$ in vector space of all polynomials, define Φ_4 by

$$\Phi_4(p) = \int_0^1 p(t^2) dt$$

(e) Let $f \in \mathbb{C}^3$ be written as (ξ_1, ξ_2, ξ_3) define functionals as below

$$(i)\Psi_1(f) = \xi_1 + \xi_2; (ii)\Psi_2(f) = \xi_1 - \xi_2 + 3; (iii)\Psi_3(f) = \xi_1 + \xi_2^3$$

[3] Let $\mathcal{V}$ be the real vector space of continuous functions $f(x)$ with $x \in [0, 1]$. Let Ψ be linear functional on the vector space $\mathcal{V}$. Let $\mathscr{M}$ be the set of all vectors $f \in \mathcal{V}$ such that $\Psi(f) = 5$. Is the set $\mathscr{M}$ a subspace of $\mathcal{V}$?

[4] Let $\mathscr{M}_1$ and $\mathscr{M}_2$ be two subspaces of a vector space $\mathcal{V}$. Show that their intersection $\mathscr{M}_1\$ \cap \mathscr{M}_2$ is also a subspace. Give two examples to show that the union is not a subspace.

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