VS-01 Problem Set

Groups, Fields, Vector Spaces and Subspaces

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[1] Consider the set of all vectors of (ξ_1, ξ_2, ξ_3) in $\mathbb{C}^3$. In which of the following cases the set of vectors form a subsapce of $\mathbb{C}^3$?

(a) ξ_1 is real. (b) $\xi_1 = 0$

(c) $|\xi_1| > 0$ (d) either ξ_1 or ξ_2 is zero

(e) $\xi_1 + \xi_2 = 0$ (f) $\xi_1 + \xi_2 = 1$

[2] Consider the set of all 3×3 real matrices A for which

(a) Tr $A = 0$ (b) det $A = 0$ (c) $A_{11} = 0$

(d) $A_{11} = A_{22} = A_{33} = 0$ (e) $A^T = A$ (f) $A^T = -A$

In which of these cases do the set of matrices A form a vector space?

[3] Consider the set of all polynomials $x(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$ for which

(a) $x(0) = 0$ (b) $2x(0) = x(1)$

(c) $x(t) = x(1-t)$ (d) $x(1) > 0$.

In which of these cases do the set of polynomials form a vector space ?

[4] Do the polynomials

$$x_1(t) = 1 - t, x_2(t) = t(1-t), x_3(t) = 1 - t^2$$

give a basis in $\mathscr{P}_2(t)$?

[5] Consider the vector space $\mathscr{P}_5(t)$ where the element $x(t)$ are polynomials of degree less than equal to 4:

$$x(t) = \alpha + \beta t + \gamma t^2 + \delta t^3 + \rho t^4$$

Let $\mathscr{M}$ be the subspace of $\mathscr{P}_5(t)$ consisting of polynomials which are even functions of t. What is the dimension of $\mathscr{M}$? What is the vector space $\mathscr{N}$ such that

$$\mathscr{P}_5(t) = \mathscr{M} \oplus \mathscr{N}$$

Give a basis in $\mathscr{N}$.

[6] In the above example what is the dimension of the quotient subspace $\mathscr{P}_5(t)/\mathscr{M}$? Give a basis for the quotient space.