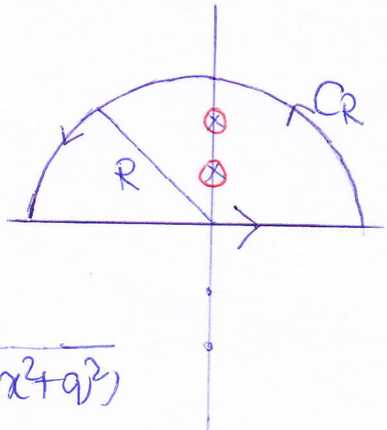
§§7.3
Q 8

$$\int_0^\infty \frac{dx}{(x^2+p^2)(x^2+q^2)} = \frac{\pi}{2pq(p+q)} \qquad p, q > 0$$

This integral can be evaluated by closing the contour in the upper half plane. So




$$\int_0^\infty \frac{dx}{(x^2+p^2)(x^2+q^2)} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dx}{(x^2+p^2)(x^2+q^2)}$$

$$= \frac{\lim}{2} \oint_{CR} \frac{dz}{(z^2+p^2)(z^2+q^2)}$$

($\lim R \to \infty$ is understood)

Only the singular points in the upper half plane need be considered. These are simple poles at $z = ip, iq$.

$$\underset{z=ip}{\text{Residue}} \frac{1}{(z^2+p^2)(z^2+q^2)} = \lim_{z\to ip} \frac{z-ip}{(z^2+p^2)(z^2+q^2)}$$

$$= \frac{1}{2ip(-p^2+q^2)}$$

$$\underset{z=iq}{\text{Residue}} = \frac{1}{2iq(p^2-q^2)}$$

$$\therefore \oint_{CR} \frac{dz}{(z^2+p^2)(z^2+q^2)} = 2\pi i \times \frac{1}{(q^2-p^2)}\left(\frac{-1}{2ip} + \frac{1}{2iq}\right)$$

$$= \frac{2\pi i}{(q^2-p^2)} \frac{(q-p)}{2ipq} = \frac{\pi}{pq(p+q)}$$

$$\therefore \int_0^\infty \frac{dx}{(x^2+p^2)(x^2+q^2)} = \frac{1}{2}\oint \cdots = \frac{\pi}{2pq(p+q)}$$

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