VS-02 Question Bank
Linear Independence, Basis and Dimension

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☹ vs-que-02001

P rove that in $\mathbb{R}^3$, three vectors $\vec{A}, \vec{B}, \vec{C}$ are linearly independent if and only if $\vec{A} \cdot \vec{B} \times \vec{C} \neq 0$.

How does this result generalize to vector space $\mathbb{C}^n$?

☹ vs-que-02002

Prove that the set of matrices

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix},$$

is a basis in the real vector space of all 2×2 real matrices.

☹ vs-que-02003

[1] Show that the three Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ do not form a basis in the complex vector space of all 2×2 complex matrices by giving an example of matrix which cannot be written as a linear combination of Pauli matrices.

[2] Describe the linear span of the three Pauli matrices in words.

☹ vs-que-02004

Do the vectors $(1, 1, 1, -1), (1, 1, -1, 1), (1, -1, 1, 1), (-1, 1, 1, 1)$ form a basis in $\mathbb{R}^4$? If your answer is 'YES' give a proof, if your answer is 'NO' give an example of a vector which is not a linear combination of these vectors.

☹ vs-que-02005

Check if the vectors $(1,1,1,-1),(1,1,-1,1),(1,-1,1,1)$, in $\mathbb{R}^4$, are linearly independent. Do they form a basis in $\mathbb{R}^4$? If your answer is 'YES' give a proof, if your answer is 'NO' give an example of a vector which is not a linear combination of these vectors.

☹ vs-que-02006

(*a*) Consider the vector space of all real numbers with field of all real numbers. Is the set $\{1,\sqrt{2}\}$ linearly independent? Does it form a basis? Give an explanation of your answer.

(*b*) Let $\mathscr{V}$ be the vector space of all real numbers with the set $\mathbb{Q}$ as the field. Is the set $\{1,\sqrt{2}\}$ linearly independent? Why? Does it form a basis?

☹ vs-que-02007

(*a*) Give an example of a basis in the complex vector space $\mathbb{C}$ over the field of all complex numbers.

(*b*) Let $\mathscr{V}$ be the vector space of all complex numbers with the set $\mathbb{R}$ as the field of scalars. Is the set $S=\{1+i,1-i\}$ linearly independent? Does it form a basis? What is the linear span of S?