QM-08 Question Bank
Canonical Quantization Using Commutators *

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Abstract

Finding matrix element between harmonic oscillator states,Ladder operators and eignefunctions of angular momentum, Constructing matrices for angular momentum operators, Zero point energy, Allowed values of j, m

☹ **qm-que-08001**

Zero point energy The potential energy of two protons in hydrogen molecule ion in a model is given below

$$V(x) = |E_1| f(x) \quad (1)$$

$$f(x) = -1 + \frac{2}{x}\left[\frac{(1-(2/3)x^2)e^{-x} + (1+x)e^{-2x}}{1+(1+x+x^2/3)e^{-x}}\right], \qquad x = R/a \quad (2)$$

$E_1 = 13.6$ eV is the ground state energy of H atom and a is the Bohr radius $\hbar^2/me^2$. The graph of this function $f(x)$ is reproduced below. Find numerical values of the bond length in A^o, the zero point energy and spacing of vibrational spectrum, both energies in electron volts.

* ver 1.x; DateCreated: Apr 14, 2015

QM-QBank-08.tex

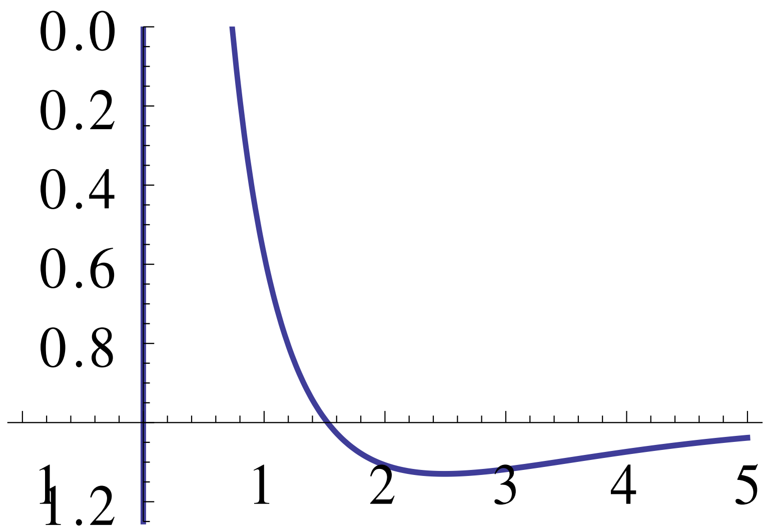

Fig. **1**

✍*The expression for $V(x)$ is taken from an approximate variational calculation of energy of the H molecule ion in Born Oppenheimer approximation*

☹ qm-que-08002

State if the combinations of j, m values, in the table given below, are allowed or not. Complete the table by writing ALLOWED/NOT ALLOWED in the second column and specifying a reason in support of your answer selecting a reason from the list, $(R1) - (R5)$, given below. In case you do not find a valid or appropriate reason listed below, feel free to select option (R6) and specify your reason.[1]

List of Possible Reasons:

(R1) All values of jm are allowed.

(R2) Not allowed because m is not an integer

(R3) Allowed because all vales of m in the range $-j$ to $+j$ are allowed

(R4) Not allowed because j, m must be an integers

(R5) Not allowed because both j, m must be integers, or half integers.

(R6) Any other reason, please specify for each case separately

☹ qm-que-08003

[1]This question requires knowledge of angular momentum eigenvalues.

Compute average value $\langle n|q^4|n\rangle$ of q^4 in the n^{th} energy state of harmonic oscillator.

☹ **qm-que-08004**

Let $|n\rangle$ denote the n^{th} excited state of a harmonic oscillator. Show that

$$\langle n|x|m\rangle = \sqrt{\frac{\hbar}{2m\omega}}\Big(\sqrt{n+1}\delta_{m,n+1} + \sqrt{n}\delta_{m,n-1}\Big)$$

☹ **qm-que-08005**

The the exact zero point energy of a several coupled oscillators is given to be $3.1415\hbar\omega_0$. What is the energy of first excited state?

☹ **qm-que-08006**

(a) What are the eigenvalues of $L^2 + \alpha L_x + \beta L_y + \gamma L_z$ for $\ell = 2$. Give a full explanation for your answer.

(b) Construct matrices for L_x, L_y, L_z for $\ell = 1$ case and verify that L^2 is a multiple of identity.

☹ **qm-que-08007**

Let $Y_{\ell m}(\theta,\phi)$ denote the simultaneous normalized eigenfunctions of L^2 and L_z operators. Use the properties of the ladder operators, $L_\pm$, and construct the expressions for $Y_{lm}(\theta,\phi)$ for $l = 2$ and $m = 2, 1, 0, -1, -2$.

Hint: $Y_{\ell\ell}(\theta,\phi)$ satisfies

$$L_z Y_{\ell\ell}(\theta,\phi) = \ell\hbar Y_{\ell\ell}(\theta,\phi) \quad (3)$$
$$L_+ Y_{\ell\ell}(\theta,\phi) = 0. \quad (4)$$

Set up these differential equations and solve to find (normalized) Y_{22}. Next apply L_- repeatedly and use

$$L_- Y_{\ell m} = \sqrt{\ell(\ell+1) - m(m+1)}\,\hbar\, Y_{\ell(m-1)}$$

to successively construct Y_{2m} for other values $m = 1, 0, -1, -2$.