Notes for Lectures in Quantum Mechanics *

Hyperfine Structure of Hydrogen Atom

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The hyperfine structure comes from the interaction of electron's dipole moment with the magnetic field due to the spin dipole moment of the nucleus. The proton, like every charged spin half particle, carries a magnetic moment given by $\mu_p = \frac{ge}{2M_p c}\vec{S}$, where g called the gyromagnetic ratio, is found to be 5.59. The magnetic moment for the proton is very small compared to the magnetic moment of the electron because the mass M_p appears in the denominator.

The hyperfine structure splitting works out to be proportional to the average value of $S_e \cdot S_p$. This operator has diagonal matrix elements between states with total spin $\vec{S} = \vec{S}_e + \vec{S}_p$. The total spin for the electron proton system can be 0 (singlet) or 1 (triplet) and we write

$$\vec{S}_e \cdot \vec{S}_p = \frac{1}{2}(S^2 S_e^2 S_p^2) \tag{1}$$

$$= \begin{cases} \frac{1}{4}, & \text{for triplet } S = 1 \\ -\frac{3}{4}, & \text{for singlet } S = 0 \end{cases} \tag{2}$$

Thus the effect of hyperfine interactions is to split each level into two levels corresponding to total spin $S = 0, 1$. Assembling all the factors, this splitting works out to be very tiny, of the order of 10^{-6} eV. For the ground state of hydrogen atom, the photon emitted in transition from triplet to singlet has the wave length of 21cm, and is the famous 21 cm line is present everywhere in radiation in the universe. That 21 cm line should be observable in emission was predicted by Dutch astronomer H. C. van de Hulst in 1944.

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