Notes for Lectures in Quantum Mechanics*

Perturbation Theory For Time Independent Problems

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Contents

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1 Basic Set up

We shall discuss the perturbation theory method for time independent problems. Given time independent Hamiltonian H, our aim is to compute the eigenvalues and eigenfunctions, ψ , of Hamiltonian and the corresponding eigenvalues W.

$$H\psi = W\psi \tag{1}$$

In the perturbation theory method the starting point is to split the Hamiltonian of the system, H, into two parts

$$H = H_0 + \lambda H'$$

Here λ is a constant.

The splitting$H = H_0 + \lambda H'$ must be done in such a way that the eigenvalue problem for H_0

$$H_0 u_n = E_n u_n \tag{2}$$

can be solved exactly.

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Both H_0 and H' will be assumed to be hermitian.

If E_n and u_n denote the eigenvalues and eigenfunctions of H_0. The eigenfunctions u_n of unperturbed Hamiltonian will be orthogonal. We will assume u_n to be normalized and hence satisfy the orthonormalization condition.

$$(u_n, u_m) = \delta_{nm} \tag{3}$$

It is further assumed that the additional part $\lambda H'$ has small in matrix elements as compared to H_0.

2 An Overview of the Method

The approximate solution is obtained by assuming that the exact eigenvalues and eigenfunctions have an expansion in powers of λ and that it is sufficient to keep first few terms of the expansion in powers of λ.

We shall write

$$H = H_0 + \lambda H' \tag{4}$$

where λ is introduced for book keeping purposes and at the end of all computations λ will be set equal to one.

Further, it will be assumed that, ψ and W can be expanded in powers of λ

$$\psi = \psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \cdots \tag{5}$$

$$W = W_0 + \lambda W_1 + \lambda^2 W_2 + \cdots \tag{6}$$

3 Order by order corrections

Substituting Eq.(5)-(6) in eigevalue equation $H\psi = W\psi$ and comparing different powers of λ gives

$$H_0\psi_0 = W_0\psi_0 \tag{7}$$

$$H_0\psi_1 + H'\psi_0 = W_0\psi_1 + W_1\psi_0 \tag{8}$$

$$H_0\psi_2 + H'\psi_1 = W_0\psi_2 + W_1\psi_1 + W_2\psi_0. \tag{9}$$

These equations are solved recursively for W_1, ψ_1, W_2, ψ_2 etc. Keeping first few terms in series expansions (8)-(9) gives the desired approximate solutions.

The terms proportional to λ^n in Eq.(5)-(6) will be called $n^{\rm th}$ order correction the eigenvalue and the eigenfunction respectively.