Examples of Flow of Energy
Charging of a Capacitor

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1. Write Anti Gray Boxes
2. Write in-text references; Directly related to the subject matter.
   Suggested textbooks; internet?
3. Write Review questions/answers
   What has been done and how ?
   How does it connect with experiments?
   How are the contents related to what you knew earlier?

Flow of energy into a capacitor being charged

When a capacitor is being charged, the energy stores is increasing with time. Taking an example of a parallel plate capacitor, we will now show that the rate of increase of energy stored is equal to the rate at which the energy flows into the volume between the plates.

Rate of increase in energy stored of the capacitor is $\frac{d}{dt}\left(\frac{1}{2}\,\frac{q^2}{C}\right) = \frac{q}{C}\left(\frac{dq}{dt}\right)$ We will now compute the flow of energy per sec from the boundary of the capacitor. Note the Poynting vector

$$\vec{N} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

has a constant magnitude every where on the curved surface $\vec{N}$ and is directed into the volume between the plates.

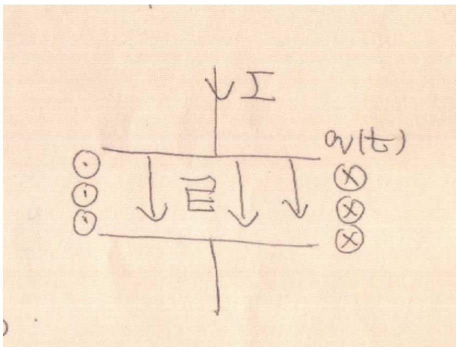

Fig. **1**

$$\begin{aligned}
E =& \frac{V}{d} = \frac{q}{Cd} \\
B =& \left(\frac{\mu_0}{2\pi a}\right) Id = \frac{\mu_0}{2\pi a} \times (Jd \times \pi a^2)
\end{aligned}$$

$\downarrow$ area of plates

$$B = \left(\frac{\mu_0}{2\pi a}\right) \times \left(\frac{\epsilon_0}{Cd}\right)\left(\frac{dq}{dt}\right) \times \pi a^2$$

Therefore, the rate of flow of energy per sec is

$$\begin{aligned}\frac{1}{\mu_0}\left|\vec{E}\times\bar{B}\right| \times \text{curved area} =& \frac{1}{\mu_0}\left|\vec{E}\times\vec{B}\right|(2\pi\ ad)\\ =& \frac{1}{\mu_0}EB\times(2\pi ad)\\ =& \frac{1}{\mu_0}\left(\frac{q}{Cd}\right)\times\left(\frac{\mu_0}{2\pi a}\right)\left(\frac{\epsilon_0}{Cd}\right)\left(\frac{dq}{dt}\right)\times\pi a^2\times(2\pi ad)\\ =& \frac{q}{c}\left(\frac{dq}{dt}\right)\left(\frac{\epsilon_0\pi a^2}{Cd}\right)\\ =& \frac{1}{2}\ \frac{d}{dt}\ \frac{q^2}{C}\end{aligned}$$

This shows that the rate of flow of energy into the volume between the plates, as given by the Poynting theorem is equal to the rate of increase of energy of the capacitor.