Computation of Electric Potential

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Abstract

The curl free nature of the electric field in electrostatics implies existence of a potential,$\phi((\vec{r}))$, from which the electric field can be derived as $\vec{E} = -\nabla\phi$. The potential at a point is just the work done in moving a unit point charge from infinity to its current position.

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1 Electric field has zero curl

It can be seen by direct computation that the electric field due to a point charge is curl free, *i.e.*it satisfies

$$\nabla \times \vec{E} = 0 \tag{1}$$

The superposition principle for the electric field then implies that (1) is obeyed by the electric field of any charge distribution.

The curl free nature means that the electrostatic forces are conservative and the work done on a unit positive charge is path independent.

2 Existence of potential

The curl free property implies, (A theorem in vector calculus) , there exists a function ϕ such that

$$\vec{E} = -\nabla \cdot \phi \tag{2}$$

If we know the electric field, the `the electric potential` ϕ can be computed by considering line integral $\int_P^Q \vec{E} \cdot \vec{dl}$, defined as a limit of a Riemann sum

$$\int_P^Q \vec{E} \cdot \vec{dl} \quad = \quad \Sigma_k E_k dl_k \cos\theta_k \tag{3}$$

θ_k= angle between tangent and $\vec{E}_k$

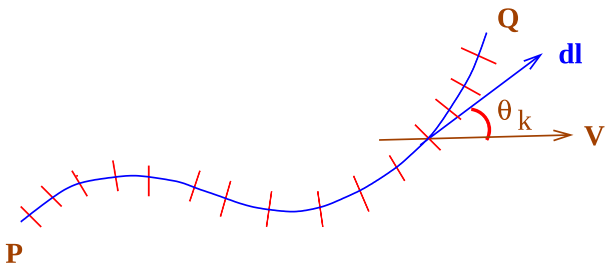

Fig. **1** Line integral as a Riemann sum

If we integrate (2), we get :

$$\int_1^2 \nabla\phi \cdot dl = \phi(\vec{r_2}) - \phi(\vec{r_1}) \tag{4}$$

Take $\vec{r}_2 \to \infty$, assuming $\phi(\vec{r}_2) \to 0$ When is this true? , and setting $\vec{r}_1 = \vec{r}$ we will get

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$$\phi(\vec{r}) = -\int_{\vec{r}}^{\infty} (\nabla\phi) \cdot \vec{dl} \tag{5}$$

or in terms of electric field :

$$\phi(\vec{r}) = \int_{\vec{r}}^{\infty} \vec{E} \cdot \vec{dl} \tag{6}$$

Thus the electric potential can be computed from the electric field. The property of electrostatic field, that the line integral is independent of the path joining the end points, is very helpful. How does it help? . Therefore, the integral in (6) can be evaluated along any convenient path. Knowing the electric field, this would give expression of the potential any charge distribution.

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