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Trigonometric and hyperbolic functions

Singular points and analytic property

1. Polynomial in z is analytic everywhere.
2. Rational functions $P(z)/Q(z)$, where $P(z), Q(z)$ are polynomials, are analytic everywhere except where $Q(z)$ is zero.
3. $\exp(\lambda z)$, $\sin(\lambda z)\cos(\lambda z)$, $\sinh(\lambda z)\cosh(\lambda z)$ are also analytic everywhere.
4. other trigonometric and hyperbolic functions have singular points as given in the table.

Functions	Singular points
$\tan z, \sec z$	$z = (2n+1)\pi/2$
$\cot z, \operatorname{cosec} z$	$z = n\pi$
$\tanh z, \operatorname{sech} z$	$z = (2n+1)i\pi/2$
$\coth z, \operatorname{cosech} z$	$z = n\pi i$

5. $\exp(\lambda z)$ does not become zero anywhere because

$$\exp(\lambda z)\exp(-\lambda z) = 1$$

So, for example, $f(z) = \frac{\sin z}{\exp(-z)}$ is analytic everywhere.