Exercise

Existence of Derivative Analyticity Property

NOTE: `This set is about relationship between existences of derivative and analytic property at a point in complex plane.`

1. Consider the following functions
$$f(z) = \exp(z)\ ,\ g(z) = \cot z$$
$$n(z) = |z|^2$$
2. Classify these three functions according to the following criteria

   Type I: Differentiable every where.

   Type II: Differentiable every where except at some *isolated points*.

   Type III: Differentiable at some *isolated* points but not differentiable at all other points.
3. Give more examples of functions of type I, II and III.
4. Sketch proof of statements
   (a) functions of type I are analytic everywhere.
   (b) functions of type II are also analytic where they are differentiable.
   (c) functions of type III are not analytic anywhere.
5. Can a function be analytic at an isolated point? If yes give an example. If NO, sketch a proof.
   NOTE: Understanding and solving the above problems should give you an idea about relation /difference between differentiability and analyticity property for functions of complex variable.
6. Give more examples of functions of Type I,II and III.

What do you learn and remember from the above questions.?
A summary is given below.

1. If a functions of one real variable is differentiable at a point, it is continuous at that point.
2. If a function of complex variable is analytic at a point z, its derivative exists at that point. In addition there exists an open disc around that point where the function is analytic.
3. The converse of the above statements is not true differentiability at a point for (complex variable)
   (a)does not imply that the functions is analytic.
   (b)does not imply that there exists an open disc around that point where the function is differentiable. In fact it may be differentiable at just that point.