Elementary Functions Existence of Derivative w,r.t. z

NOTE: In this set we will take a look about existence of derivative w.r.t. complex variable z.

1. The functions c^x, $\sin x$, $\cos x$ are *continuous* and *differentiable* every where. WHY?
   Simplest way to see is that for these functions derivative exists everywhere in the real line. Therefore they are continuous on the real line.
2. take exponential of a complex variable $\exp(z)$. Its real and imaginary parts are given by

$$\begin{aligned}\exp(z) =& e^x(\cos y + i\sin y)\\ u(x,y) =& e^x\cos y, \qquad v(x,y) = e^x \sin y\end{aligned}$$

3. Do the functions $u(x,y)$, and $v(x,y)$ satisfy the Cauchy Riemann equations.
4. Find partial derivations

$$\frac{\partial u}{\partial x}, \frac{\partial V}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial V}{\partial y}$$

   Are these continuous functions of x, y?
5. The answers to $Q3$ and $Q4$ is YES. Therefore, e^z is differentiable at all points.
6. The statement (5) implies that $\sin z$, $\cos z$, $\sinh z$, $\cosh z$ are all differentiable for all z.
7. $\tan z$, see z are differentiable everywhere except where $\cos z = 0$.
8. $\tanh z$, $\cosh z$ are differentiable everywhere except where $\cosh z = 0$.
9. $\cot z$, cosec$z$ are differentiable except where $\sin z$ is zero because

$$\cos z = \frac{\cos z}{\sin z}, \ \text{cosec} z = \frac{1}{\sin z}$$

10. $\coth z$, cosec hz are differentiable everywhere except where $\sinh z = 0$ because

$$\coth z = \frac{\cosh z}{\sinh z}, \ \text{cosec h} z = \frac{1}{\sinh z}$$