Notes for Lectures in Quantum Computation *

Entanglement

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Definition 1 *State of a physical system is called* **pure state** *if it is represented by a vector in Hilbert space of all states.*

Definition 2 *If it is given that the probability of system being in a set of states* $\{|\psi_k\rangle\}$ *are* $\{p_k| \sum_{p>0} p_k = 1\}$*. We say that such a state is a* **mixed state.**

A mixed state is described by density matrix convex linear combination,

$$\rho = \sum_{k=1}^{n} p_k |\psi_k\rangle\langle\psi_k| \tag{1}$$

of projection operators $|\psi_k\rangle\langle\psi_k|$. For a pure state a particular state $|\psi\rangle$ has $p = 1$ and the corresponding density matrix is $\rho = |\psi\rangle\langle\psi|$.

Definition 3 *A system consisting to two subsystems* A, B *is described by a vector in the tensor product of Hilbert spaces* $\mathcal{H}_A \otimes \mathcal{H}_B$*. In general a state of composite system has the form*

$$|\Psi\rangle = \sum_{k=1}^{n} C_K |\psi_{AK}\rangle \tag{2}$$

A state is **separable** *if the state vector has only one term in (2) i.e.* $|\Psi\rangle$ *can be written as a direct product*

$$|\Psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \tag{3}$$

for some $|\psi_A\rangle$ *and* $|\psi_B\rangle$ *in Hilbert spaces* $\mathcal{H}_A$ *and* $\mathcal{H}_B$*.*

Definition 4 *We say that the system is in an* **entangled state** *if the state vector cannot be written as product (3). i.e. the state vector is not separable.*

Bell States: *For a two level system, Bell states are given by*

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle \pm |1,1\rangle)$$
$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle \pm |1,0\rangle)$$

These states are entangled states and play an important role in quantum information theory.

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Definition 5 *We will now define a* **mixed entangled state:** *If the density operator ρ of a composite system can be written as*

$$\rho = \sum p_i \rho_A^i \otimes \rho_B^i$$

on Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$, we say that the mixed state is separable; otherwise the system is in non-separable, or mixed entangled state.