Notes for Lectures in Quantum Computing and Quantum Information *

Binary Number Representation

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

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1 Representation in binary numbers

Binary representation uses a sequence of 0 and 1 to represent a given positive integer. We use the notation that roman alphabets will take values 0,1. Thus $x \in \{0, 1)$, similarly $a_1, a_2, \cdots, b_1, \cdots$ etc., will take values in $\{0, 1\}$. These numbers will be called bits, or classical bits, or cubits.
A sequence of bits such as

$$a_n \cdots\cdots a_1 a_0 \equiv a$$

will be called multi bit and represents the number

$$N = a_n 2^n + a_{n-1} 2^{n-1} + \cdots a_2\ 2 + a_0$$

For example, a 3 bit number 111 is

$$\begin{aligned} 111 \mapsto 1 \times 2^2 + 1 \times 2^1 + 1 &= 7 \\ 11001 \mapsto 1 \times 2^5 + 1 \times 2^3 + 1 &= 32 + 8 + 1 = 41 \end{aligned}$$

A sequence of n classical bits represents numbers in the range 0 to $2^n - 1$.
For example, using 6 bits, we can represent any integer in the range 0 to $2^6 - 1 = 63$.
To represent a number p, at least $\log_2 p$ bits are needed.
For example, to represent 1000 as a binary number at least $\log_2 1000 = 9.97 \to 10$ bits are required.

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2 Boolean algebras

Several binary operations can be performed on two classical bits.
These operations form elements of an algebra known as Boolean algebra

$x + 1 = 1$	$A \cdot \bar{A} = 0$
$x + 0 = x$	$A + B = B + A$
$x \cdot 1 = x$	$\overline{A + B} = \bar{A} \cdot \bar{B}$
$x \cdot 0 = 0$	$\overline{A \cdot B} = \bar{A} + \bar{B}$
$x + x = x$	$x \cdot x = x$
$x + \bar{x} = 1$	$x \cdot y = x \wedge y$
NOT $x = \bar{x}$	$x + y = x \vee y$
NOT (NOT x)=x	NOT $\bar{x}$=x

One should think of bits 1 and 0 as true and false respectively.

3 Basic operations

- **AND:** x AND y also written as $x \wedge y$ satisfies

$$x \wedge y = \begin{cases} 1 & \text{if } x = y = 1 \\ 0 & \text{otherwise} \end{cases}$$

- **OR:** x OR y, also written as $x \vee y$ satisfies

$$x \vee y = \begin{cases} 0 & \text{if } x = y = 0 \\ 1 & \text{otherwise} \end{cases}$$

- **NOT:** NOT x, denoted by $\neg x$, is defined by

$$\neg x = \begin{cases} 0 & \text{if } x = 1 \\ 1 & \text{if } x = 0 \end{cases}$$

Question 1 *It is correct to say that we can represent the above binary operations on numbers 0 and 1, as arithmetic operations modulo 2.*
So the question is,"Is it consistent to use the following?"
$x \wedge y = x \cdot y$ *(multiplication mod 2);*
$x \vee y = x + y$, *(addition modulo 2); and*
$\neg x = 1 - x$.

4 Properties of binary operations

Distributive property
$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
$x \vee (y \cap z) = (x \vee y) \wedge (x \vee z)$
Associative property
$x \cup (y \cup z) = (x \cup y) \cup z$
$x \cap (y \cap z) = (x \cap y) \cap z$

Double negation

$\neg(\neg x) = x$

Commutative property

$x \vee y = y \vee x$

$x \wedge y = y \wedge x$

Question 2 *What is being represented in terms of binary numbers? Integers? Rational numbers? Irrational numbers? Any real number?*

Reference: Wikipedia