Problem Set 3 [Solution]

7. $$S = \frac{n}{2}\left[c_1 + 5R \ln \frac{U}{N} + 2R \ln \frac{V}{N}\right]$$

$$\left.\frac{\partial S}{\partial U}\right)_{V,N} = \frac{1}{T} = \frac{5nR}{2U} \qquad U = \frac{5}{2} nRT$$

$$\left.\frac{\partial S}{\partial V}\right)_{U,n} = \frac{p}{T} = \frac{nR}{V} \qquad pV = nRT$$

(a) $$S = \frac{n}{2}\left[c_1 + 5R \ln \left[\frac{5}{2} RT\right] + 2R \ln\left[\frac{V}{N}\right]\right]$$

$$T \left.\frac{\partial S}{\partial T}\right)_V = T \frac{n}{2} \cdot \frac{(5R)}{T} = \frac{5}{2} nR.$$

$$S = \frac{n}{2}\left[c_1 + 5R \ln\left[\frac{5}{2} RT\right] + 2R \ln \left[\frac{RT}{P}\right]\right]$$

$$= \frac{n}{2}\left[c_1 + 7R \ln T - 2R \ln P + \text{const}\right]$$

$$C_p = T \left.\frac{\partial S}{\partial T}\right)_P = \frac{7nR}{2}.$$

(b) The outside and inside pressure is the same in both the situations. So is the volume of the room.

$$V_1 = V_2$$
$$P_1 = P_2$$

Thus $$U = \frac{5}{2} V P.$$

$$\frac{U_1}{V_1} = \frac{U_2}{V_2} = \frac{5}{2} P$$

8. The change in entropy (Final temperature T_3)

$$\Delta S = C_V \int_{T_1}^{T_3} \frac{dT}{T_1} + C_V \int_{T_2}^{T_3} \frac{dT}{T} = C_V \ln\left[\left(\frac{T_3}{T_1}\right)\left(\frac{T_3}{T_2}\right)\right] \geq 0$$

or $T_3^2 \geq T_1 T_2$ [equality for a reversible engine]

Maximum work is when $\Delta S = 0$ or $T_3 = \sqrt{T_1 T_2}$

9. $$W = C_V (T_1 - T_3) + C_2 (T_2 - T_3) = C\,(T_1 + T_2 - 2\sqrt{T_1 T_2}) = C\left(\sqrt{T_1} - \sqrt{T_2}\right)^2.$$

Problem Sheet. 4

Solution

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$$\tau = aT\left[\frac{L}{L_0(T)} - \frac{L_0^2(T)}{L^2}\right]$$

$(F = U - TS)$

$$\Delta Q = dU - \underbrace{\int_{L_0(T)}^{L} \tau\, dL}_{\text{W.D on the band}}$$

$\tau\, dL + \Delta Q = dU.$

$$= dU + aT\left[\frac{L^2}{2L_0(T)} + \frac{L_0^2}{L}\right] - dU_0 - \frac{3}{2}aTL_0$$

= Answer to (a) ..

$\tau\, dL + \Delta Q = dU. \Rightarrow T\,dS = dU - \tau\, dL$

(b) $dF = dU - TdS - SdT$

$= \tau\, dL - S\,dT$

(c) $\left.\frac{\partial F}{\partial L}\right)_T = \tau = aT\left(\frac{L}{L_0(T)} - \left(\frac{L_0}{L}\right)^2\right);$

$$\left.\begin{matrix} F(T,L) \\ -F(T,L_0) \end{matrix}\right\} = aT\left[\frac{L^2}{2L_0(T)} + \frac{L_0^2}{L}\right] - \frac{3}{2}aTL_0(T)$$

$$\left.\frac{\partial F}{\partial T}\right)_L = -S(T,L) = a\left[\frac{L^2}{2L_0(T)} + \frac{L_0^2}{L} - \frac{3}{2}aL_0\right]$$

$$+ aT\left[-\frac{L^2}{2L_0^2} + \frac{2L_0}{L}\right]\frac{dL_0}{dT} - \frac{3}{2}aT\frac{dL_0}{dT} + \underbrace{\frac{\partial F(T,L_0)}{\partial T}}_{-S(T,L_0)}$$

$S(T,L) - S(T,L_0)$

$$= -a\left[\frac{L^2}{2L_0} + \frac{L_0^2}{L} - \frac{3}{2}L_0\right]$$

$$- a\frac{dL_0}{dT}T\left[-\frac{L^2}{2L_0^2} + \frac{2L_0}{L} - \frac{3}{2}\right]$$

$$= a\left[\frac{3}{2}L_0 - \frac{L^2}{2L_0} - \frac{L_0^2}{L}\right]$$

$$- a\frac{dL_0}{dT}T\left[\frac{L^2}{2L_0^2} - \frac{2L_0}{L} + \frac{3}{2}\right]$$

11. (a) T is const

$$\Delta Q = \cancel{T\left[S(T,L_0) - S(T,L)\right]}\; T\left[S(T,L) - S(T,L_0)\right]$$

$$= aT\left[\frac{3}{2}L_0 - \frac{L^2}{2L_0} - \frac{L_0^2}{L}\right] + aT\frac{dL_0}{dT}\left[\frac{L^2}{2L_0^2} - \frac{2L_0}{L} + \frac{3}{2}\right]$$

(b)

$$\left.\frac{\partial T}{\partial L}\right)_S = -\frac{\left.\frac{\partial T}{\partial S}\right)_L}{\left.\frac{\partial L}{\partial S}\right)_T} = -\frac{\left.\frac{\partial S}{\partial L}\right)_T}{\left.\frac{\partial S}{\partial T}\right)_L}$$

$$\left.\frac{\partial S}{\partial L}\right)_T = a\left[-\frac{L}{L_0} + \frac{L_0^2}{L^2}\right] - a\frac{dL_0}{dT}T\left[\frac{L}{L_0^2} + \frac{2L_0}{L^2}\right]$$

$$\left.\frac{\partial S}{\partial T}\right)_L = \frac{C_L}{T}$$

$$\left.\frac{\partial T}{\partial L}\right)_S = -\frac{T}{C_L}\left[a\left(-\frac{L}{L_0} + \frac{L_0^2}{L^2}\right) - aT\frac{dL_0}{dT}\left[\frac{L}{L_0^2} + \frac{2L_0}{L^2}\right]\right]$$

$$\left.\frac{\partial T}{\partial L}\right)_S = \frac{aTL_0^2}{C_L L^2}\left[-1 + \frac{L^3}{L_0^3} + \frac{Ta}{L_0}\frac{dL_0}{dT}\left[\frac{L^3}{L_0^3} + 2\right]\right]$$

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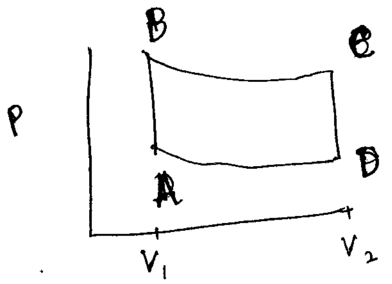

$P_A V_1 = RT_A$ $P_B V_1 = RT_B$; $P_C V_2 = RT_C$; $P_D V_2 = RT_D$

U_{AB} = Heat given to the system [as $\Delta W_{AB} = 0$]

$= \cancel{V_1[P_A - P_D]}\; V_1[P_B - P_A]$

No heat in BC & AD

U_{CD} = Heat given by the system $= V_2[P_C - P_D]$

$$\eta = 1 - \frac{V_2[P_C - P_D]}{V_1[P_B - P_A]} \qquad \frac{P_A}{P_B} = \frac{P_D}{P_C} = \lambda$$

we also have

$$P_B V_1^\gamma = P_C V_2^\gamma$$
$$P_A V_1^\gamma = P_D V_2^\gamma$$

$$\eta = 1 - \frac{V_2 P_C[1-\lambda]}{V_1 P_B[1-\lambda]} = 1 - \frac{V_2 P_C}{V_1 P_B}$$

$$\eta = 1 - \frac{V_2}{V_1}\left(\frac{V_1^\gamma}{V_2^\gamma}\right) = 1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$