5th Oct

Problem Sheet ~~I~~ due ~~19 Sept.~~ 2021
Solution

4 (a) $S = K_1 (NVU)^{1/3}$. $K_1 > 0$

Postulate II satisfied $N, U, V \to \lambda N, \lambda U, \lambda V$
$S \to \lambda S$.

$$\left.\frac{\partial S}{\partial U}\right)_{V,N} = \frac{1}{3} K \frac{(NV)^{1/3}}{U^{2/3}} > 0 \quad \text{Postulate III}$$

$$U = \left(\frac{S}{K_1}\right)^3 \frac{1}{NV}$$

If $\left.\frac{\partial U}{\partial S}\right)_{V,N} = 0 \Rightarrow S = 0$ Postulate IV satisfied.

(b) $S = K_2 \left(\frac{NU}{V}\right)^{2/3}$

does not satisfy $N \to \lambda N$, $U \to \lambda U$, $V \to \lambda V$
$S \to \lambda S$. Postulate III not satisfied.

(c) $S = K_3 \frac{V^3}{NU}$

$\left.\frac{\partial S}{\partial U}\right)_{V,N} \propto -\frac{1}{U^2}$ is not positive Postulate III not satisfied.

(d) $S = N \ln \left[\frac{UV}{N^2 K_4}\right]$

$$\frac{VU}{N^2 K_4} = e^{S/N} \quad ; \quad U = \frac{N^2 K_4}{V} e^{S/N}$$

$$\left.\frac{\partial U}{\partial S}\right)_{V,N} = \frac{N K_4}{V} e^{S/N} \Rightarrow 0 \quad S \to -\infty.$$

Not valid.

Solutions Problem sheet 2

5) The final pressure in all the three chambers are equal. $P_1 = P_2 = P_3$

The Temperature in the two changes A_2 and A_3 are equal $T_2 = T_3 = \frac{9T_0}{4}$

$(A_2 + A_3)$ is adiabatic

$$P_2 (V_2 + V_3)^\gamma = P_0 (2V_0)^\gamma \qquad V_2 = V_3 = \frac{RT_2}{P_2} = \frac{RT_3}{T_3}.$$

$$P_2 V_2^\gamma = P_0 V_0^\gamma \qquad P_0 V_0 = RT_0$$

$$P_2 V_2 = RT_2$$

$$\cancel{P_1 = P_0 \left(\frac{V_0}{V_2}\right)^\gamma = P_0 \left(\frac{4}{9}\right)^{5/3}} \qquad \cancel{\frac{V_0}{V_2} = \left(\frac{T_0}{T_2}\right) = \frac{4}{9}}$$

$$\cancel{= P_0 \left(\frac{4}{9}\right)^\gamma} \qquad \gamma = 5/3$$

$$P_2^{1/\gamma} V_2 = P_0^{1/\gamma} V_0 \qquad \frac{1}{\gamma} - 1 = \frac{3}{5} - 1 = -\frac{2}{5}$$

$$\frac{P_2^{1/\gamma} T_2}{R P_2} = \frac{P_0^{1/\gamma} T_0}{R P_0}$$

$$P_2^{\left(\frac{1}{\gamma}-1\right)} T_2 = P_0^{(1/\gamma - 1)} T_0$$

$$P_2^{-2/5} T_2 = P_0^{-2/5} T_0$$

$$P_2^{-2/5} = P_0^{-2/5} \left(\frac{4}{9}\right)$$

$$P_2 = P_0 \left(\frac{4}{9}\right)^{-5/2} = P_0 \left(\frac{2}{3}\right)^{-5} = P_0 \left(\frac{32}{243}\right)^{-1} = P_0 \left(\frac{2}{3}\right.$$

$$V_2 = \frac{RT_2}{P_2} = \frac{R\left(\frac{9T_0}{4}\right)}{P_0 \left(\frac{32}{243}\right)^{-1}} = V_0 \left(\frac{9}{4}\right) \times \frac{32}{243}$$

$$V_2 = \frac{8}{27} V_0 \qquad \begin{array}{r} 81 \\ 16 \\ \hline 65 \end{array}$$

Thus $P_f = \frac{243}{32} P_0$ $\qquad V_2 = V_3 = \frac{8}{27} V_0$

$$V_1 = \left(3 - \frac{16}{27}\right) V_0 = \frac{65}{27} V_0.$$

$$T_1 = \left(\frac{243}{32}\right)\left(\frac{65}{27}\right) \frac{P_0 V_0}{R} = \frac{585}{32} T_0$$

Work done by A_1 = Increase in energy of $(A_2 + A_3)$

W.D on $(A_2 + A_3)$

$V_2 + V_3$ = Constant.

Thus $\Delta U = 2 C_v \Delta T = 2 \times \frac{3}{2} \times R \left[\frac{9T_0}{4} - T_0 \right]$

$= 3R \times \frac{5}{4} T_0 = \frac{15}{4} R T_0 = \frac{15}{4} P_0 V_0.$

Heat supplied equals the total increase in energy.

$\Delta Q = \Delta U_{A_1} + \frac{15}{4} P_0 V_0$

$\Delta U_{A1} = 1 \times \frac{3}{2} R \times \left[\frac{585}{32} - 1 \right] T_0$

$\Delta Q = \left[\frac{3}{2} \left(\frac{553}{32} \right) + \frac{15}{4} \right] P_0 V_0$

$= \left(\frac{1659}{64} + \frac{15}{4} \right) P_0 V_0 = \frac{1899}{64} P_0 V_0$

$585 - 32 = 553$

$553 \times 3 = 1659$

$\frac{1659 + 180}{64}$

$1659 + 180$