Notes for Lectures in Quantum Mechanics *

Einstein A and B Coefficients

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1 Introduction

The semiclassical theory gives a reasonably good description of atomic transitions induced by a light incident on an atom. It fails to give an explanation for observed spontaneous emission of radiation when an atom is in an excited state. Einstein showed how to compute the transition probability for induced emission using Planck's law of black body radiation. This work was done in 1917 much before even quantum mechanics arrived.

2 Spontaneous Emission

We assume that atoms are placed inside a cavity. When the atoms are in an equilibrium with radiation at temperature T, the number of photons of a particular frequency emitted per sec must be the same as the number of photons absorbed per sec by the atoms. Note that the probability of absorption per unit time can be computed using semiclassical theory. Let's call probability per unit time as A.

$$A = \frac{4\pi e^2}{3\hbar^2 c} I(\omega) |\langle f|\hat{e}\cdot\vec{r}|i\rangle|^2 \qquad (1)$$

The probability of induced emission per unit time is given by the same expression,

Let N_1 and N_2 denote the number of atoms in levels E_1 and E_2 with $E_2 > E_1$. When in equilibrium at temperature T, the number of atoms in the level with energy E_k, will be proportional to the Boltzmann factor

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number $\exp(-E_k/kT)$. Let B denote the transition probability per unit time for the induced emission to take place in the state E_2.

We therefore have The total number of transitions from E_1 to E_2 (absorption)
$= N_1\times$ Probability per unit time for induced absorption
The number of transitions from E_2 to E_1 (emission)
$= N_2\times$(probability per unit time of induced emission
$+$ probability per unit time of spontaneous emission)

This gives

$$\exp(-E_1/k_BT) \times A = \exp(-E - 2/k_BT)(A + B). \tag{2}$$

Substituting (1) for A using Planck's Law for the frequency distribution $I(\omega)$

$$I(\omega) = \frac{\hbar\omega^3}{\pi^2 c^2 \Big(\exp\big(\hbar\omega/k_BT\big) - 1\Big)} \tag{3}$$

Solving for B gives

$$B = \frac{4e^2\omega^2}{3\hbar c^3}|\langle\vec{r}\rangle|^2 \tag{4}$$

which is the required transition probability per unit time for spontaneous emission.