

Problem Set on Lie Algebras

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Abstract

- ⊙ Answer the following questions for $SU(2)$ and $SU(3)$.
- [1] What is the Lie algebra ? What is the dimension of the Lie algebra? Do the anti hermitian matrices belong to the Lie algebra?
 - [2] Give the number of elements in the Cartan subalgebra. How many raising (lowering) generators are present? Are these E_α elements of the Lie algebra?
 - [3] Give the number of roots and list all root vectors. Which of the roots are positive roots and which ones are simple roots.
 - [4] What is the dimension of adjoint representation? Specify one place where the adjoint representation makes its appearance in particle physics.
 - [5] What are the fundamental representations? How many fundamental representation are there? Give a few examples where these appear in particle physics applications.
 - [6] Draw root diagram showing all the root vectors.
 - [7] Sketch the weight diagrams for (i) fundamental representation(s) and (ii) adjoint representation.
 - [8] Taking $SU(2)$ as isospin and $SU(3)$ as flavour group, give examples of particle multiplets that fit into
 - fundamental representation
 - adjoint representation
 - [9] For the $SU(3)$ of standard model, is it correct that the elements $\{H_1, H_2\}(\equiv \{F_3, F_8\})$ of Cartan subalgebra are proportional to the 3rd component of isospin and hypercharge Y ?
 - [10] Construct the spin matrices for spin $3/2$.
 - [11] Construct the $SU(3)$ generators for 3 and 3^* representations of $SU(3)$. How do these compare with Gell-Mann matrices for $SU(3)$.
(**Hint:** Use commutation rules given in Gasiorowicz, p261, 262)