Notes For Lectures in Quantum Mechanics *

Charged Particle in E.M. Field

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

Contents

---

1 Introduction

We will describe the interaction of an electron in an atom with e.m. waves in a semiclassical fashion. The electron will will be treated quantum mechanically, whereas the e.m. waves will be treated classically. We shall ignore the effect of motion of the electron on the e.m. waves. We will interested in finding out what are allowed transitions of an electron that can take place in presence of e.m. waves and we want to compute the corresponding transition probabilities.The effect of other electrons will be represented by a potential $V(\vec{r})$ while the scalar and vector potentials $\phi(\vec{r}), \vec{A}(\vec{r})$ will describe the effect of the e.m. waves.

We shall take the Hamiltonian of the electron in an atom to be given by

$$H = \frac{1}{2m}\Big(\vec{p} - \frac{e}{c}\vec{A}\Big)^2 + q\phi(\vec{r}) + V(\vec{r}) \tag{1}$$

*Updated:Jul 17, 2021; Ver 0.x

We expand the first term and rewrite the Hamiltonian as

$$\begin{aligned} H &= \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + q\phi(\vec{r}) + V(\vec{r}) && (2) \\ &= \frac{\vec{p}^2}{2m} - \frac{e}{2mc}\big(\vec{A}\cdot\vec{p} + \vec{p}\cdot\vec{A}\big) + \frac{e^2}{2mc}\vec{A}^2 + q\phi(\vec{r}) + V(\vec{r}) && (3) \\ &= \frac{\vec{p}^2}{2m} - \frac{e}{2mc}\big(2\vec{A}\cdot\vec{p} + -i\hbar\nabla\vec{A}\big) + \frac{e^2}{2mc}\vec{A}^2 + q\phi(\vec{r}) + V(\vec{r}) && (4) \end{aligned}$$

where in the last step use has been made of the commutator

$$\vec{p}\cdot\vec{A} - \vec{A}\vec{p} = -i\hbar\nabla\cdot\vec{A} \qquad (5)$$

2 Coulomb Gauge

We shall work in the gauge

$$\phi = 0 \qquad \nabla\cdot\vec{A} = 0 \qquad (6)$$

In this gauge the Hamiltonian H takes the form

$$\begin{aligned} H &= H_0 + H' && (7) \\ H_0 &= \frac{\vec{p}^2}{2m} + V(\vec{r}) && (8) \\ H' &= \frac{e}{mc}\vec{A}\cdot\vec{p} + \frac{e^2}{2mc}\vec{A}^2 && (9) \end{aligned}$$

where H_0 describe the electron in the atom, H'

$$H' = \frac{e}{mc}\vec{A}\cdot\vec{p} + \frac{e^2}{2mc}\vec{A}^2 \qquad (10)$$

gives the effect of e.m. waves on the electron and will be treated as a perturbation.

3 Weak field approximation

The interaction Hamiltonian H' will be spilt as as $H' = H_1 + H_2$ with

$$H_1 = \frac{e}{mc}\vec{A}\cdot\vec{p}, \qquad H_2 = \frac{e^2}{2mc}\vec{A}^2 \qquad (11)$$

It will be seen that the term H_2 is of higher order as compared to H_1 and can be ignored in the first order calculations.